Additional Notes on vectors

Suppose you have two points in space:

$$(x_1, y_1) \& (x_2, y_2)$$

(1) Construct the vectors pointing from the origin to each point.

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} : \vec{r}_2 = x_2\hat{i} + y_2\hat{j}$$

(2) Find the magnitude of each of these vectors.

$$|\vec{r}_1| = \sqrt{\vec{r}_1 \bullet \vec{r}_1} = \sqrt{x_1^2 + y_1^2} : |\vec{r}_2| = \sqrt{\vec{r}_2 \bullet \vec{r}_2} = \sqrt{x_2^2 + y_2^2}$$

(3) Construct the unit vector for each of these vectors.

$$\hat{r}_1 \equiv \frac{\vec{r}_1}{|\vec{r}_1|} = \frac{x_1\hat{i}+y_1\hat{j}}{\sqrt{x_1^2+y_1^2}} : \hat{r}_2 \equiv \frac{\vec{r}_2}{\sqrt{x_2^2+y_2^2}} = \frac{x_2\hat{i}+y_2\hat{j}}{\sqrt{x_2^2+y_2^2}}$$

(4) Construct the vector pointing from point 1 towards point 2.

$$\vec{r}_{ip} \equiv \vec{r}_p - \vec{r}_i$$

You should read this as "the vector pointing from point i towards point p."
It looks like this in 3-dimensions:

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = \left[x_p - x_i\right]\hat{i} + \left[y_p - y_i\right]\hat{j} + \left[z_p - z_i\right]\hat{k}$$

For the present 2 vectors at hand:

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = \left[x_2\hat{i} + y_2\hat{j}\right] - \left[x_1\hat{i} + y_1\hat{j}\right] = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

(5) Find the magnitude of the vector pointing from point 1 towards point 2.

$$|\vec{r}_{12}| = \sqrt{\vec{r}_{12} \bullet \vec{r}_{12}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(6) Find the unit vector pointing from point 1 towards point 2.

$$\hat{r}_{12} \equiv \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

(7) Find the angle between $\hat{r}_{12}$ and the x, and then the y axis.

$$\cos(\theta_x) = \frac{\hat{r}_{12}\bullet\hat{i}}{|\hat{r}_{12}||\hat{i}|} = \frac{(x_2-x_1)}{\left[(x_2-x_1)^2+(y_2-y_1)^2\right]} : \theta_x = \cos^{-1}(\theta_x) \text{ or } \theta_x = 360 - \cos^{-1}(\theta_x)$$

$$\cos(\theta_y) = \frac{\hat{r}_{12}\bullet\hat{j}}{|\hat{r}_{12}||\hat{j}|} = \frac{(y_2-y_1)}{\left[(x_2-x_1)^2+(y_2-y_1)^2\right]} : \theta_y = \cos^{-1}(\theta_y) \text{ or } \theta_y = 360 - \cos^{-1}(\theta_y)$$

Notice that since $\cos(\varphi) = \cos(360-\varphi) = \cos(-\varphi)$, you will need to look at the values of the components to get the actual angles. To find the actual angle, you have to look at the quadrant in which the vector lies but there is an easy way to calculate it:

(a) Find θ_x. If the y-component is less than zero, the actual angle is 360-θ_x.

(b) Find θ_y. If the x-component is greater than zero, the actual angle is 360-θ_y.

(8) Evaluate $\vec{r}_1 + \vec{r}_{12} - \vec{r}_2$

$$\vec{r}_1 + \vec{r}_{12} + \vec{r}_2 = \left[x_1\hat{i} + y_1\hat{j}\right] + \left[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}\right] - \left[x_2\hat{i} + y_2\hat{j}\right]$$

$$\Rightarrow \vec{r}_1 + \vec{r}_{12} + \vec{r}_2 = \left[x_1 - x_1 + x_2 - x_2\right]\hat{i} + \left[y_1 - y_1 + y_2 - y_2\right]\hat{j} = \vec{0}$$

Which means that if you go around a complete circle, you end up where you started.

I have made a spreadsheet that will allow you to try this out with different coordinates.