Monday, Sept. 25, 2006	Physics 240: Worksheet 11	Name ________________

(1) Suppose a mass m slides down a frictionless inclined plane (of angle θ). At the bottom of the plane, the mass encounters a flat surface with a coefficient of friction μ. How far does the mass move beyond the bottom of the plane if it falls through a vertical height y? Provide numerical answers for the case y=1m and μ=0.3.

(2) A cyclist approaches the bottom of a hill at a speed of 11 m/s. The hill is 6 m high. Ignoring friction, how fast is the cyclist moving at the top of the hill? What is the meaning of the general solution if the hill is 9 m high? :)

(3) **Using energy considerations**, what is the speed of a rock when it has fallen through a distance of 100 m if it started from rest?

(4) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal.

(5) Now, what is there was a coefficient of friction μ=0.4 between the mass and the table which it is sliding on in problem 4. If the mass was moving at 15 m/s when it encountered the spring, how far will the spring compress?

Monday, Sept. 25, 2006	Physics 240: Worksheet 11	Name ________________

(1) Suppose a mass m slides down a frictionless inclined plane (of angle θ). At the bottom of the plane, the mass encounters a flat surface with a coefficient of friction μ. How far does the mass move beyond the bottom of the plane if it falls through a vertical height y? Provide numerical answers for the case y=1m and μ=0.3.

You want to apply energy conservation here. The equation is then:

$$\Delta K_{NC} = \Delta K + \Delta U$$

Now, looking at the problem from beginning to end, you see that in fact we do have the case: $\Delta K = 0$

We also have:

$$\Delta K_{NC} = W_f = -\mu mg(\Delta x)$$

We also for the gravitational potential energy have:

$$\Delta U = -mgy$$

So let's put it all together:

$$\Delta K_{nc} = \Delta K + \Delta U \Rightarrow -\mu mg(\Delta x) = -mgy \Rightarrow \Delta x = \tfrac{y}{\mu}; \Delta x = \tfrac{1}{.3} = 3.33m$$

(2) A cyclist approaches the bottom of a hill at a speed of 11 m/s. The hill is 6 m high. Ignoring friction, how fast is the cyclist moving at the top of the hill?
Solution: Total mechanical energy is conserved here. Thus, $\Delta U + \Delta K = 0$. This problem is a bit unlike other problems since the kinetic energy is not zero at any time in this problem. We need to calculate each of the relevant terms here. The terms are: $\Delta U = mgy - 0 = mgy$ and $\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$. Let's now write the energy conservation equation:

$$\Delta K + \Delta U = 0 \Rightarrow \left[\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right] + [mgy - 0] = 0$$

No you want to solve this for the final velocity. Thus:

$$\left[\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right] = -mgy \Rightarrow \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 - mgy \Rightarrow v_f^2 = v_i^2 - 2gy$$

The solution for the final velocity is then:

$$v_f = \pm\sqrt{v_i^2 - 2gy}$$

For this particular numerical example, we then get: $v_v = \pm\sqrt{v_i^2 - 2gy} = \pm\sqrt{121 - 2(9.8)6}$.
We are only asked for "how fast" in this problem so we choose the positive sign. This give us the velocity of $v_f = \sqrt{3.4} = 1.84 m/s$.

(3) Using energy considerations, what is the speed of a rock when it has fallen through a distance of 100 m if it started from rest?
Solution: Total mechanical energy is conserved here. Thus, $\Delta U + \Delta K = 0$. We need to calculate each of the relevant terms here. $\Delta U = -mgy - 0 = -mgy$ and $\Delta K = \tfrac{1}{2}mv^2 - 0 = \tfrac{1}{2}mv^2$. Thus, $-mgy + \tfrac{1}{2}mv^2 = 0 \Rightarrow v = \pm\sqrt{2gy}$. The – solution is the physical solution here. Putting in the values, we find: $v = -\sqrt{2(9.8)100} = -\sqrt{1960} = -44.3 m/s$.

(4) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal.

This problem conserves total mechanical energy. Thus: $\Delta U + \Delta K = 0$. We need to calculate each of the relevant terms. $\Delta U = \frac{1}{2}k\left[x_f^2 - x_i^2\right]$ and $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$. Let's put everything together. I'm going to assume the spring is initially at x_i=0.

$$\Delta U + \Delta K = 0 \Rightarrow \Delta U = \tfrac{1}{2}k\left[x_f^2 - x_i^2\right] - \tfrac{1}{2}mv^2 = 0 \Rightarrow x_f = \pm\sqrt{\tfrac{m}{k}}v$$

The final position of the mass is then:

$$x_f = \pm\sqrt{\tfrac{m}{k}}v$$

But in this problem, we're asked "how much will the spring compress" so we're looking for Δx. The symbolic problem solution is then:

$$\Delta x = \pm\sqrt{\tfrac{m}{k}}v$$

If the mass is initially traveling in the +x direction, then the physical solution is the positive

$\frac{1}{2}kx^2 - \frac{1}{2}mv^2 = 0$. Solve this for x: $x = \pm\sqrt{\frac{m}{k}}v$. I'll assume the mass is moving in the +x direction initially. Then the physical solution is +. Let's put in the values to find then

$$\Delta x = \sqrt{\frac{100\,\text{kg}}{5\frac{\text{N}}{\text{m}}}}15\tfrac{\text{m}}{\text{s}} = \sqrt{\frac{100\,\text{kg}}{5\frac{\text{kg m/s}^2}{\text{m}}}}15\tfrac{\text{m}}{\text{s}} = \left[\sqrt{20\,\text{s}^2}\right]15\tfrac{\text{m}}{\text{s}} = 67.1\,\text{m}$$

(5) Now, what if there was a coefficient of friction μ=0.4 between the mass and the table which it is sliding on in problem 4. If the mass was moving at 15 m/s when it encountered the spring, how far will the spring compress?

Solution: We need to include the loss of kinetic energy due to non-conservative forces in this problem because of friction. The more general form is then:

$$\Delta K_{NC} = \Delta U + \Delta K$$

We need to evaluate each of these terms.

$$\Delta K_{NC} = \vec{f}\bullet\vec{x} = -\mu mg\left[\Delta x\right] = -\mu mgx_f,\ \Delta U = \tfrac{1}{2}k\left[x_f^2 - x_i^2\right] = -\tfrac{1}{2}kx_f^2 \text{ and } \Delta K = -\tfrac{1}{2}mv^2.$$

We can put it all together to obtain:

$$-\mu mgx_f = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}mv^2.$$

I believe that you will find it easier at this point to put in as many numerical values and then re-express the equation as a standard quadratic. Thus,

$$-\mu mg = -.4(100)9.8 = -392.\ \tfrac{1}{2}k = 2.5 \text{ and } \tfrac{1}{2}mv^2 = 50(15)^2 = 11250.$$

We put these into the quadratic to obtain:

$$-392\left[x_f\right] = 2.5\left[x_f\right]^2 - 11250.$$

Let's put this in standard form:

$$2.5\left[x_f\right]^2 + 392\left[x_f\right] - 11250 = 0 \Rightarrow \left[x_f\right]^2 + 156.8\left[x_f\right] - 4500 = 0.$$

This is solved by the quadratic formula:

$$x_f = \frac{-156.8\pm\sqrt{(156.8)^2 - 4(1)(-4500)}}{2(1)} = \frac{-156.8\pm\sqrt{(156.8)^2 + 18000}}{2} = \frac{-156.8\pm 206.4}{2} = \begin{matrix} +24.8 \\ -181.6 \end{matrix}.$$

We can see that if the mass is initially traveling in the +x direction, the spring must compress also in the +x direction. Thus the positive solution is correct here. Incidentally, the negative solution here would never be correct even if the mass were moving in the –x direction initially.

The physical solution here needs to be positive (this is actually defined not by v but by the way we calculated the work against friction). Thus, x_f=+24.8 m.

What happens if $\mu = .04$? Then the quadratic equation looks like $x^2 + 15.68x - 4500 = 0$.

It's pretty easy to solve this: $x_f = \frac{-15.68 \pm \sqrt{(15.68)^2 + 18000}}{2} = \frac{-15.68 \pm 135.1}{2} = \begin{matrix} 59.71\,\text{m} \\ -75.4\,\text{m} \end{matrix}$.

The solution here is then x=+59.7 m.

$$\text{What if } \mu = 0\text{? } x_f = \frac{-0 \pm \sqrt{(0)^2 + 18000}}{2} = \frac{\pm 135.1}{2} = \begin{matrix} +67.1\,\text{m} \\ -67.1\,\text{m} \end{matrix}$$

which is the same as in problem 4. Why go through these extra steps? This also gives a bit more insight into the non-physical solution … it's the solution that results if things work the wrong way in a sense (friction would speed things up, which is not what happens). I suppose that the moral to this story is that if you are not sure about which solution is the physical solution, manipulate the input variables till they reflect what is the solution from a simpler problem, if possible.

Let me now show you one more little detail. Return to problem 4:
(6) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal and the spring is initially compressed 30 m.

The thing to remember about springs is this: the more you push, the harder it pushes back. With this reasoning, the final position ought not to be 97.1 m. Let's see what the answer is.

This problem conserves total mechanical energy. Thus: $\Delta U + \Delta K = 0$. We need to calculate each of the relevant terms. $\Delta U = \frac{1}{2}k\left[x_f^2 - x_i^2\right]$ and $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$. Let's put everything together. I'm going to assume the spring is initially at x_i=30m.

$$\Delta U + \Delta K = 0 \Rightarrow \frac{1}{2}k\left[x_f^2 - x_i^2\right] - \frac{1}{2}mv^2 = 0 \Rightarrow x_f^2 = x_i^2 + \frac{m}{k}v \Rightarrow x_f = \sqrt{x_i^2 + \frac{m}{k}v^2}$$

The final position of the spring is then:

$$x_f = \sqrt{30^2 + \frac{100}{5}(15)^2} = \sqrt{900 + 4500} = \sqrt{5400} = 73.5\,\text{m}$$

So, the answer here for the change in length is $\Delta x = 43.5\,\text{m}$

So although the final compression is more, the change in length was less, in keeping with the general observation that the more you push, the more it pushes back.