

Instructions: You have a total of 50 minutes to complete this test.

Answer each of the following questions completely.

Time Start _____ Time finish _____ Pledged _____

You must supply all details that led to your answer. You must provide correct SI units where required.

Do not discuss any aspect of this test with anyone until I return the test.

Although you may use additional sheets of paper which should be turned in with your test, please write (neatly) your answers on the pages where the problems are presented.

(1) Three vectors are given by: $\vec{A} = 3\hat{i} + 10\hat{j}$, $\vec{B} = 10\hat{i} - 3\hat{j}$, and $\vec{C} = -5\hat{i} + 5\hat{j}$.

(a) What is the angle between vectors \vec{B} and \vec{C} ?

$$\vec{B} \cdot \vec{C} = -50 - 15 = -65 = (10.44)(7.07) \cos(\theta) \Rightarrow \cos(\theta) = -\frac{65}{(10.44)(7.07)} = -0.881$$

$$\Rightarrow \theta = 151.7^\circ \text{ also: } 208.3^\circ$$

(b) What is the value of $\vec{A} \cdot \vec{B}$?

$$\vec{A} \cdot \vec{B} = 30 - 30 = 0$$

(c) Suppose a particle moves through vector \vec{A} and then through vector \vec{B} . What is the vector pointing towards the final position of the particle?

$$\vec{D} = \vec{A} + \vec{B} = (3+10)\hat{i} + (10-3)\hat{j} = 13\hat{i} + 7\hat{j}$$

(d) What is the value of $\vec{B} - 2\vec{C}$?

$$\vec{B} - 2\vec{C} = (10+10)\hat{i} + (-3-10)\hat{j} = 20\hat{i} - 13\hat{j}$$

(2) A rocket is observed photographically to have a position vector given by:

$$\vec{R} = 30t\hat{i} + 2t^5\hat{j} \text{ m during the first 5 seconds of flight.}$$

(a) What is the velocity **vector** of the rocket at any time during the first 5 seconds?

$$\vec{v} = \frac{d\vec{R}}{dt} = 30\hat{i} + 10t^4\hat{j} \frac{\text{m}}{\text{s}}$$

(b) What is the acceleration vector of the rocket at any time during the first 5 seconds?

$$\vec{a} = \frac{d\vec{v}}{dt} = 0\hat{i} + 40t^3\hat{j} \frac{\text{m}}{\text{s}^2}$$

(c) provide numerical answers, together with correct SI units at $t=5$ s.

$$\vec{v} = 30\hat{i} + 6250\hat{j} \text{ m/s}$$

$$\vec{a} = 0\hat{i} + 5000\hat{j} \frac{\text{m}}{\text{s}^2}$$

(3) You have been promoted to become the commander and lead gunner of the UEDAA (United Earth Defenses Against Asteroids). Satellite telemetry indicates that an incoming asteroid will pass right over your position (on the Earth) at an altitude of 10000 m traveling with a constant velocity vector of $\vec{v} = 1000\hat{i} + 0\hat{j} \frac{\text{m}}{\text{s}}$.

(a) What must be the velocity of a projectile which is fired from your gun so that it can reach this altitude at its maximum position? You may ignore wind resistance.

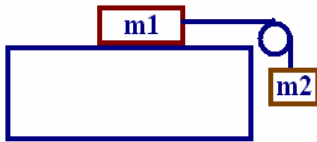
$$v_y^2 = v_{y,0}^2 - 2g(\Delta y) \Rightarrow v_{y,0} = \sqrt{2gh} = 442.7 \text{ m/s}$$

(b) How long will it take the projectile to reach this altitude?

$$v_y = v_{0,y} - gt \Rightarrow t = \frac{v_{0,y}}{g} = \frac{442.7}{9.8} = 45.2 \text{ s}$$

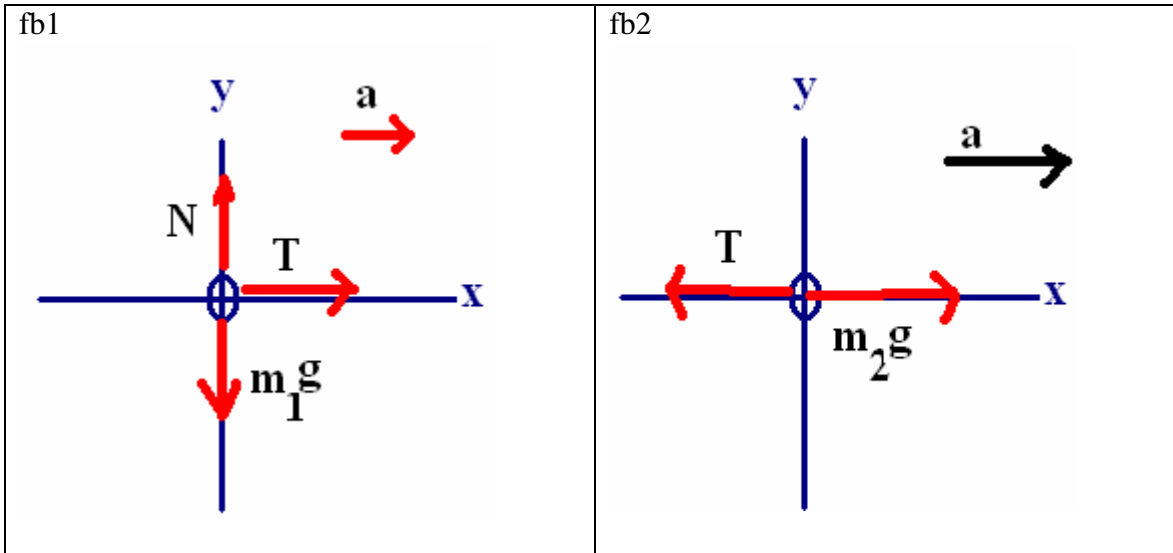
(c) What was the position **vector** of the asteroid at the time which the projectile was fired if the projectile is to strike the asteroid with the initial velocity obtained in part (a)?

$$x = -v_{0,x}t = -45200 \text{ m} \Rightarrow \vec{R} = -45200\hat{i} + 10000\hat{j} \text{ m}$$



(4) Two masses are connected by a string as shown. Mass m_1 is resting on a frictionless table.

(a) Provide complete and correct free body diagrams for the system. “unbend”



(b) Find the acceleration of the system (symbolically).

$$\sum \vec{F} = m\vec{a}$$

fb1:

$$y: N - m_1g = 0 \Rightarrow N = m_1g$$

$$x: T = m_1a$$

fb2:

$$x: m_2g - T = m_2a$$

$$m_2g - m_1a = m_2a \Rightarrow m_2g = (m_1 + m_2)a \Rightarrow a = g \frac{m_2}{(m_1 + m_2)}$$

(c) Find the tension in the string (symbolically).

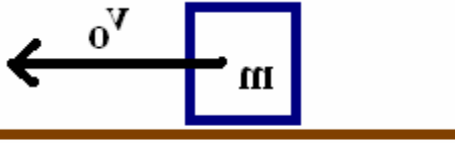
$$T = m_1a = g \frac{m_1m_2}{(m_1 + m_2)}$$

(d) Suppose $m_1=1\text{kg}$ and $m_2=2\text{kg}$. Provide numerical answers to (b) and (c) with correct SI units.

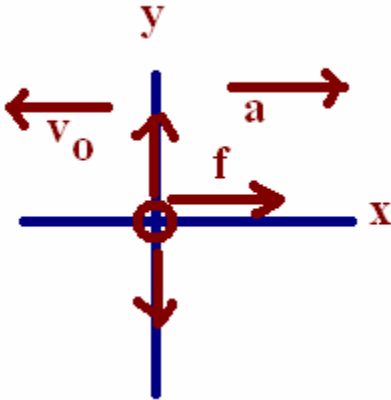
$$a = g \frac{m_2}{(m_1 + m_2)} = g \frac{2}{3} = 6.53 \frac{\text{m}}{\text{s}^2} : T = 6.53 \text{ N}$$

(5) A mass m is resting on a flat surface as shown. The coefficient of kinetic friction between the mass and the surface is μ . The mass is then kicked so that it has an initial velocity v_0 .

(a) Draw a complete free body diagram of the system, also showing the direction of the acceleration.



reverse the image so the acceleration is in the +x direction.



(b) Find the acceleration of the system, symbolically.

$$x : f = ma$$

$$\sum \vec{F} = m\vec{a} \Rightarrow y : N - mg = 0 \Rightarrow f = \mu mg = ma \Rightarrow a = \mu g$$

$$f = \mu N$$

(c) How far does the system move until it stops, symbolically?

$$v^2 = v_0^2 + 2a(\Delta x) \Rightarrow \Delta x = -\frac{v_0^2}{2a}$$

(d) How long does it take until the system stops, symbolically?

$$v = v_0 + at \Rightarrow t = -\frac{v_0}{a}$$

(e) Provide numerical results, together with correct SI units for the case $\mu = 0.4$, and $|v_0| = 5 \frac{m}{s}$

$$a = \underline{\hspace{2cm}} \quad \Delta x = \underline{\hspace{2cm}} \quad t = \underline{\hspace{2cm}}$$

$$a = \underline{3.92 m/s^2} \quad \Delta x = \underline{-3.19 m} \quad t = \underline{1.23 s}$$