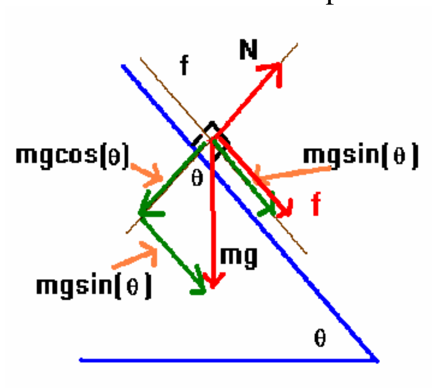
Special inclined plane problem

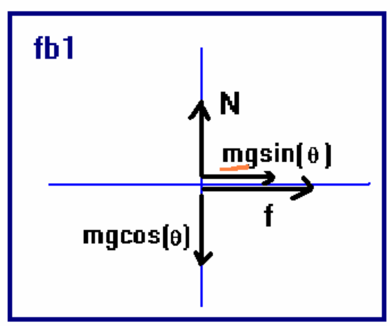
Suppose the inclined plane has a coefficient of friction μ, is tilted at the angle θ and the mass is given an initial velocity v_0 up the plane. How long does it take for the mass to fall off of the plane if the mass is initially at a height d and how fast is it moving at the bottom of the plane.

This problem needs to be solved in 2 parts. First, let's find out how far up the plane the mass moves.

This is our construction for the first part of the problem:



Next we draw the free body diagram:



next we apply Newton's laws:

$$\sum \vec{F} = m\vec{a}$$

We thus have:

$$y: N - mg\cos(\theta) = 0 \Rightarrow N = mg\cos(\theta)$$

$$x: f + mg\sin(\theta) = ma$$

and the frictional force is:

$$f = \mu N = \mu mg\cos(\theta)$$

we thus have the conclusion:

$$\mu mg\cos(\theta) + mg\sin(\theta) = ma \Rightarrow a = g\left[\mu\cos(\theta) + \sin(\theta)\right]$$

Now let's find the displacement along the direction of the initial velocity:

$$v^2 = v_0^2 + 2g\left[\mu\cos(\theta) + \sin(\theta)\right](\Delta x) \Rightarrow \Delta x = -\frac{v_0^2}{2g\left[\mu\cos(\theta)+\sin(\theta)\right]}$$

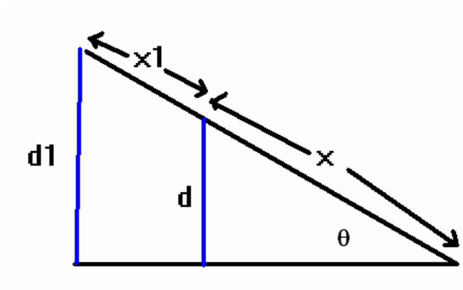
This tells us how far up the plane the mass slides.

Now, let's find how long it took to slide up this far:

$$v = v_0 + at \Rightarrow t = -\frac{v_0}{a} = -\frac{v_0}{[\mu\cos(\theta)+\sin(\theta)]}$$

Now it is useful to find the altitude above the base of the plane:

$$d = x\sin(\theta); (x_1 + x)\sin(\theta) = d_1 \Rightarrow d_1 = \left[\frac{v_0^2\sin(\theta)}{2g[\mu\cos(\theta)+\sin(\theta)]}\right] + d$$



where I have now calculated the total distance from the bottom of the plane.
(it involved dropping a minus sign).

Now for the rest of the problem … the mass is a distance d1 above the ground and it is going to be moving with a velocity given by:

$$v = \sqrt{2gd_1} = \sqrt{2g\left[\left[\frac{v_0^2\sin(\theta)}{2g[\mu\cos(\theta)+\sin(\theta)]}\right] + d\right]}$$

at the bottom of the plane (look at problem 2 on worksheet 8).
How long does this take?

$$t_2 = \frac{v_x - v_{0,x}}{a_x} = \frac{\sqrt{2gd}}{a_x} = \frac{\sqrt{2g\left[\left[\frac{v_0^2\sin(\theta)}{2g[\mu\cos(\theta)+\sin(\theta)]}\right]+d\right]}}{g\sin(\theta)}$$

(see worksheet 8, problem #2 again).
Thus the total time is given by:

$$t_{total} = t + t_2 = -\frac{v_0}{[\mu\cos(\theta)+\sin(\theta)]} + \frac{\sqrt{2g\left[\left[\frac{v_0^2\sin(\theta)}{2g[\mu\cos(\theta)+\sin(\theta)]}\right]+d\right]}}{g\sin(\theta)}$$

It looks nasty but the point is this problem is quite doable using the techniques that I have taught you.