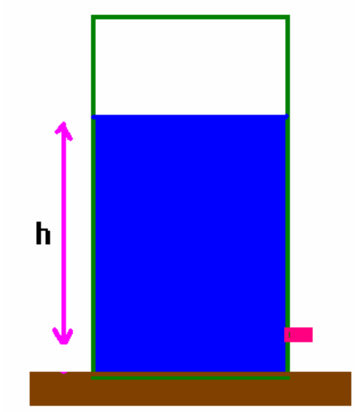
	Physics 210: Worksheet 31	Name: ________________

(1) Consider a tank as shown. There is a small hole near the bottom. Suppose that the tank is not so high so that the atmospheric pressure makes a big difference from the top to the bottom.



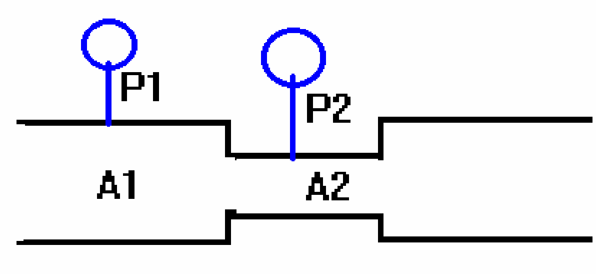
(a) If the tank is open to the atmosphere at the top, how fast does the fluid flow out?

(b) If the top of the tank has a pressure P greater than the atmospheric pressure, how fast does the fluid flow out?

(c) Suppose that the tank was closed at the top but initially did not have an over pressure. When would the fluid stop flowing out of the tank?

(d) How far can the atmosphere lift a column of water?

(2) A tube consists of the elements shown below. Show how this can be used to find fluid velocity if the fluid has a density ρ.



(3) Suppose you eat lots of bad food that eventually blocks the cross sectional area of an artery by a factor of ½. How much lower is the blood pressure in this region of the artery than an unblocked region?

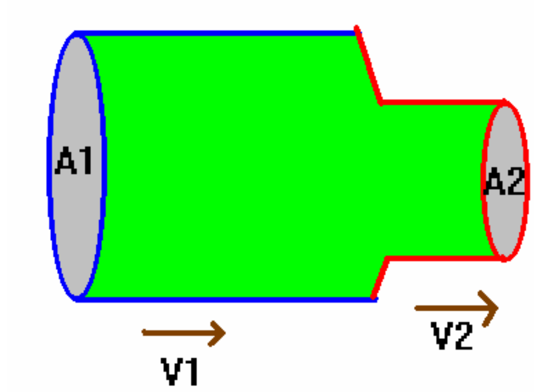
(4) Suppose a plunger has a cross sectional area of 0.1 m^2 while a second plunger has an area of 0.01m^2. The system is connected together via yucky orange fluid as shown before. How much weight will the large plunger be able to pick up if 10 kg is placed on the small plunger?

Bernoulli's Equation, Pascal's Principle and Fluid dynamics

Note: Our fluids that we will discuss are

(1) irrotational (2) incompressible and (3) frictionless.

At this level, there are 3 important principles that need to be understood. Let's first obtain the continuity equation.



Consider the pipe section shown above. The pipe cross sectional area changes from A1 to A2. The question here is what happens to the fluid velocity.

If the pipe is not leaky and the fluid does not compress, then:

Fluid in = Fluid out

we can calculate each of these sides. Also since the fluid does not compress, the density is constant throughout the fluid. Thus: in a time Δt, we have:

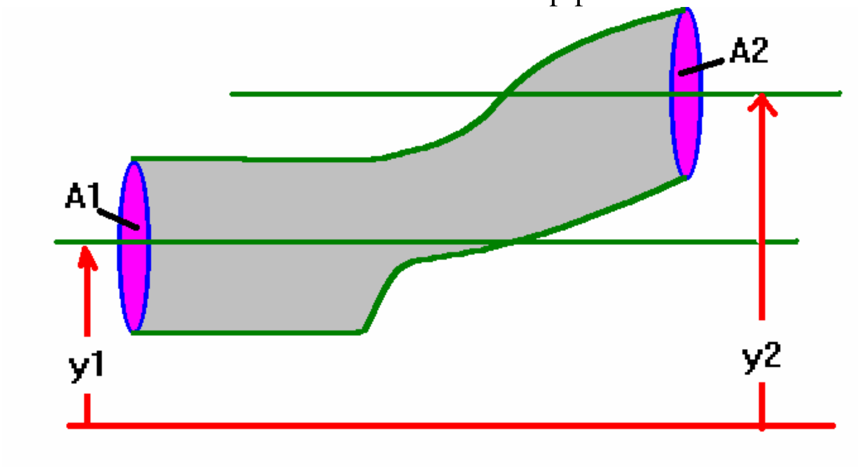
$$\rho A_1\left(v_1\left(\Delta t\right)\right) = \rho A_2\left(v_2\left(\Delta t\right)\right)$$

This is easily solved to give:

$$v_2 = \frac{A_1}{A_2} v_1$$

This is known as the “continuity equation” for fluids.

Next, you may have wondered just how fluid flow works.
Let's consider another pipe:



Here, I've made the cross sectional area constant for this pipe. The important thing here is the pipe is kind-of bent so that one end is at a different level than the other.

The pressure at one end does work on the fluid. From the work-energy theorem:

$$W = \Delta K + \Delta U$$

What is doing this work is a pump of some kind somewhere along the line ***but certainly external to the fluid under consideration***. In any event, we can relate this to pressure by:

$$W = (P_1 - P_2)A(\Delta x)$$

We can write the other terms as:

$$\Delta K = K_2 - K_1$$

and

$$\Delta U = U_2 - U_1$$

Ok, let's put it all together:

$$(P_1 - P_2)A(\Delta x) = K_2 - K_1 + U_2 - U_1$$

Let's collect the 1's and 2's on separate sides of the equation:

$$P_1 A(\Delta x) + K_1 + U_1 = P_2 A(\Delta x) + K_2 + U_2$$

Now we what to write out K and U in different forms:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left[\rho A(\Delta x)\right]v^2$$

$$U = mgy = \left[\rho A(\Delta x)\right]gy$$

Clearly, we'll be able to divide by the volume term A(Δx) to give:

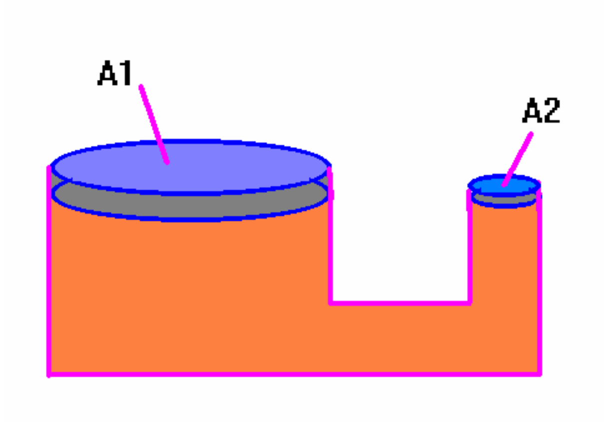
$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

to simplify this, we have:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = \text{constant}$$

This very important equation is known as Bernoulli's Equation and follows directly from the conservation of energy.

The final new topic I need to introduce in order to make your intro to fluid dynamics complete is Pascal's principle.



Consider the contraption I've sketched above. It consists of 2 plungers connected to a u-shaped tube filled with a yucky orange fluid. What we do here is to apply a small overpressure to one of the areas. What it does to the other area is given by Pascal's principle which states that ***the pressure pulse is transmitted undiminished through the fluid***.

Let's see how this all works: If you place a weight say m_1 on plate A_1, then the pressure pulse is given by:

$$\Delta P_1 = \frac{m_1 g}{A_1}$$

According to Pascal's principle, this overpressure must be the same at plate 2. Thus, at plate 2,

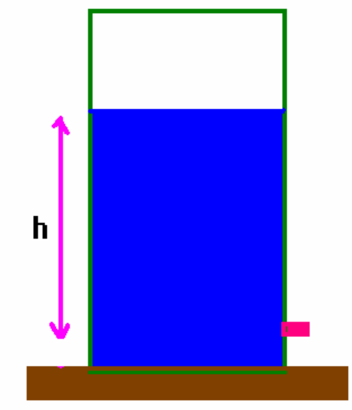
$$\Delta P_2 = \Delta P_1$$

This overpressure will lift a weight m2. How much will it lift?

$$\frac{m_1 g}{A_1} = \frac{m_2 g}{A_2} \Rightarrow m_2 = m_1 \left(\frac{A_2}{A_1}\right)$$

The essence is that the smaller A_1 is, the bigger the mass m_2 that can be lifted. This is basically how a hydraulic lift or a hydraulic press works.

(1) Consider a tank as shown. There is a small hole near the bottom. Suppose that the tank is not so high so that the atmospheric pressure makes a big difference from the top to the bottom.



(a) If the tank is open to the atmosphere at the top, how fast does the fluid flow out?
(b) If the top of the tank has a pressure P greater than the atmospheric pressure, how fast does the fluid flow out?
(c) Suppose that the tank was closed at the top but initially did not have an over pressure. When would the fluid stop flowing out of the tank?
(d) How far can the atmosphere lift a column of water?

From Bernoulli's equation, we have:

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \text{constant}$

(a) Call the top of the tank (1) and the bottom of the tank (2). If the tank is open and not too tall, then P_1 is about the same as P_2 *and is due to the* ***external*** *atmosphere*. Next, notice that the fluid flow at the top of the tank is not significant so we'll set v_1 to zero. Thus, we have:

$$\rho g\left(y_1 - y_2\right) = \frac{1}{2}\rho v_2^2 \Rightarrow v_2 = \sqrt{2gh}$$

This is the speed of efflux of the fluid from the tank and is exactly the same as the velocity that a ball will have when it falls through a height h.

(b) In the second case, with the pressure P greater than atmospheric pressure (retaining all the other approximations), we have:

$$P + \rho g h = \frac{1}{2}\rho v_2^2 \Rightarrow v_2 = \sqrt{2\left(gh + \frac{P}{\rho}\right)}$$

(the fluid squirts out faster if you squeeze on it).
This is basically Torcelli's Equation.

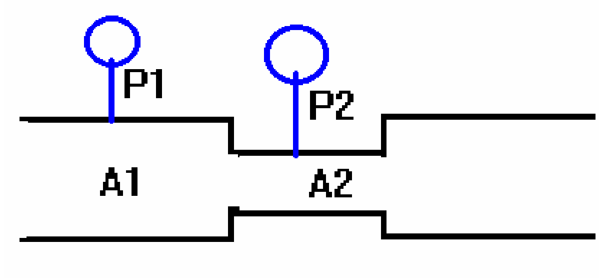
(c) In this case, everything is the same as in part (b) except that now P<0. The fluid stops flowing when v_2=0. Thus, for the fluid to stop flowing, we require:
$P + \rho g h = 0 \Rightarrow P = -\rho g h$. The fluid stops flowing when the underpressure is equal to the pressure at the bottom of the column of water or the atmospheric pressure supports the column of water.

(d) How far can a vacuum lift a column of water?
Here, we essentially imagine P_1=0, v_1=v_2=0 and P_2=1x10^5 Pa. Thus, we have the result:

$$0 + \rho g h = P_{atm} = 1\text{x}10^5\,\text{Pa} \Rightarrow h = \frac{1\text{x}10^5\,\text{Pa}}{1\text{x}10^3\frac{\text{kg}}{\text{m}^3}\text{x}9.8\frac{\text{m}}{\text{s}^2}} = 10.2\text{m}$$

The length of a mercury column with a density of about 10x that of water is on the order of 1 m.

(2) A tube consists of the elements shown below. Show how this can be used to find fluid velocity if the fluid has a density ρ.



Solution: From the continuity equation, we have:

$$v_2 = v_1\left(\frac{A_1}{A_2}\right)$$

Now use this in Bernoulli's equation, keeping the altitudes the same:

$$\left[P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2\right] \Rightarrow P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_1^2\left(\frac{A_1}{A_2}\right)^2$$

$$\tfrac{1}{2}\rho v_1^2 - \tfrac{1}{2}\rho v_1^2\left(\frac{A_1}{A_2}\right)^2 = \left[P_2 - P_1\right] \Rightarrow \tfrac{1}{2}\rho v_1^2\left[1 - \left(\frac{A_1^2}{A_2^2}\right)\right] = \Delta P$$

Solve this for v_1:

$$v_1 = \sqrt{\frac{2\Delta P}{\rho\left(1 - \frac{A_1^2}{A_2^2}\right)}}$$

This is how oil companies might measure the velocity of fluid flow in an oil pipeline.

(3) Suppose you eat lots of bad food that eventually blocks the cross sectional area of an artery by a factor of ½. How much lower is the blood pressure in this region of the artery than an unblocked region?
Solution: The continuity equation tells us how much faster the fluid flows in the blocked region. Let 1 be unblocked and 2 be blocked. Then:

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) \Rightarrow v_2 = v_1\left(\frac{A_1}{\frac{1}{2}A_1}\right) = 2v_1$$

so the fluid is moving also twice as fast. We can calculate the pressure differential between the two areas by Bernoulli's equation:

$$\left[P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2\right] \Rightarrow P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho\left[4v_1^2\right] \Rightarrow \Delta P = \tfrac{3}{2}\rho v_1^2$$

The blocked region would have a pressure decrease.
There will be a bulge before the blocked area putting stress on the walls. After the block, the blood will squirt like a cutting spray against the artery walls. You might imagine that the density of blood is about that of water. You can also get an idea of how fast the blood might be flowing by measuring the height of a person. Hu?
ok, here is that last part: The pressure difference between the feet and the head is about

$\Delta P = \rho g h$ and it's this pressure difference that makes the blood squirt from the feet to the head. It will need a velocity of about $\rho g h = \frac{1}{2}\rho v^2 \Rightarrow v = \sqrt{2gh}$

This can be used to place a numerical value on the pressure difference if you like. For a 2 meter tall person, v=6.3 m/s so $\Delta P \approx \frac{3}{2}[1000][6.3]^2 \approx 6\times10^4\ \text{Pa}$. Of course, this last part could be way off owing to my approximations.

(4) Suppose a plunger has a cross sectional area of 0.1 m^2 while a second plunger has an area of 0.01m^2. The system is connected together via yucky orange fluid as shown before. How much weight will the large plunger be able to pick up if 10 kg is placed on the small plunger?

Solution: let 1 be the smaller plunger and let 2 be the larger plunger. Then, according to Pascal's principle:

$$\frac{m_1 g}{A_1} = \frac{m_2 g}{A_2} \Rightarrow m_2 = m_1\left(\frac{A_2}{A_1}\right)$$

The ratio of areas is a factor of 10. Thus,

$$m_2 = 10\left(\frac{10}{1}\right) = 100\text{kg}$$