	Physics 240:Worksheet 05	Name ________________

(1) Suppose a squirrel is hanging from a tree. The distance from the ground is Y_t m. A hunter is in a foxhole located a distance X_t from the tree and the barrel of the gun is at ground level. When the gun makes a noise, the squirrel drops from the tree (always). Where should the hunter point the gun to be 100% sure of hitting the squirrel under these conditions?

(2) A boat is moving along a river with a constant velocity $\vec{V}_{boat} = V_{xb}\hat{i} = +5\hat{i}\,\frac{m}{s}$. A bridge is over the river and on the bridge is one of those mean little boys that you always read about for having done bad things. This particular boy holds a rock at a distance of 20m above the water. How long before the front of the boat is directly under the boy will the boy need to release the object that he is going to release so that it hits the front of the boat?

(3) A hunter is hunting clay targets which are thrown straight up into the air with an initial velocity $V_{t,o}$. Provided the hunter is a distance x which is close enough to the throwing machine, can the hunter always hit the targets by pointing at them?

(4) The Earth has an average radius of 6.37×10^6m. In order for an object to orbit the Earth, as a minimum it must cross a distance equal to the radius of the Earth in the time that it takes it to fall through the distance of the radius of the Earth (assuming the Earth is spherical). What is the minimum velocity which an object can have to orbit the Earth?

(5) It's fill-er-up time at the local dairy bar! Cowboy Ryan is seen busy behind the bar filling glasses of everyone's favorite malt beverage[1] and then sliding the glasses down to waiting customers. One of the customers missed catching the glass and it slid right off that bar and smashed onto the ground a distance of 4m from the base of the bar which was 1.5 m high. How fast was the glass moving when it left the bar?

[1] chocolate malt.

(1) Suppose a squirrel is hanging from a tree. The distance from the ground is Y_t m. A hunter is in a foxhole located a distance X_t from the tree and the barrel of the gun is at ground level. When the gun makes a noise, the squirrel drops from the tree (always). Where should the hunter point the gun to be 100% sure of hitting the squirrel under these conditions?

Solution: The equation of motion for the bullet is

$$Y_b = Y_i + V_{y,i}t - \tfrac{1}{2}gt^2 \text{ and } X_b = X_i + V_{x,i}t\ .$$

The equation of motion for the squirrel is

$$Y_t = Y_{ti} - \tfrac{1}{2}gt^2 \text{ and } X_t = X_{ti}\ .$$

We want to solve for the intersection of these equations:

$$Y_b = Y_t \text{ and } X_b = X_t$$

in order to say the bullet hit the squirrel (the times are the same since the squirrel lets go when the gun shoots). For the Y intersection, we have:

$$Y_b = Y_t \Rightarrow Y_{bi} + V_{yb,i} - \tfrac{1}{2}gt^2 = Y_{ti} - \tfrac{1}{2}gt^2$$

and for the X intersection, we have:

$$X_b = X_t \Rightarrow X_{bi} + V_{xb,i}t = X_t\ .$$

Since the gun is at y=0, and assuming that it is also at x=0, we have the following two equations:

$$Y_{ti} - \tfrac{1}{2}gt^2 = V_{yb,i}t - \tfrac{1}{2}gt^2 \text{ and } V_{xb,i}t = X_t\ .$$

The Y equation becomes simpler:

$$Y_{ti} = V_{yt,i}t\ /$$

We can divide these two equations to eliminate t:

$$\frac{Y_t}{X_t} = \frac{V_{yt,i}t}{V_{xb,i}t} = \frac{V_{yt,i}}{V_{xb,i}} = \tan(\theta)$$

where θ is the angle that the gun should be pointed at. So the conclusion is this: to hit the squirrel, point the gun at the squirrel. This will remain to be true so long as the gun is close enough so that the squirrel does not hit the ground before the intersection occurs.

(2) A boat is moving along a river with a constant velocity $V_{boat} = V_{xb}\hat{i} = 5\hat{i}\ \mathrm{m/s}$. A bridge is over the river and on the bridge is one of those mean little boys that you always read about for having done bad things. This particular boy holds a rock at a distance of 20m above the water. How long before the front of the boat is directly under the boy will the boy need to release the object so that it hits the front of the boat? Where was the boat at this time?

Solution: Let the x coordinate of the boy be x=0. The initial y position of the object is y_o=20 m. Thus the relevant equations of motion are:

$$X_b = X_{bi} + v_{xb}t;\ Y_b = 0;\ X_o = 0;\ \text{and } Y_o = Y_{oi} - \tfrac{1}{2}gt^2$$

We are looking for the intersection. From the Y equations, we obtain: $t = \pm\sqrt{\frac{2Y_{oi}}{g}} = \pm 2.02\text{s}$. Here, the physical solution is negative: t=-2.02s. We can then find out where the boat was at this time. We know that at t=0, the boat is at x=0. Thus, $X_b = 0 + V_{xb}t = -5(2.02) = -10.1\text{ m}$

(3) A hunter is hunting clay targets which are thrown straight up into the air with an initial velocity $V_{t,o}$. Provided the hunter is a distance x which is close enough to the throwing machine, can the hunter always hit the targets by pointing at them?

Solution: For the targets: $x = x_t$ and $y = y_{to} + V_{to,y}t_1 - \frac{1}{2}gt_1^2$ while for the bullet, the equations of motion are $x = x_o + V_{bo,x}t_2$ and $y = y_{bo} + v_{bo,y}t_2 - \frac{1}{2}gt_2^2$. For an intersection, we then have

$$x_t = x_o + v_{bo,x}t_1 \text{ and } y_{bo} + v_{bo,y}t_1 - \tfrac{1}{2}gt_1^2 = y_{to} + v_{to,y}t_2 - \tfrac{1}{2}gt_2^2.$$

Now, we can let everything correspond to the time that the target hunter pulls the trigger. How? It's because this simply requires a restatement of the initial target position and velocity. Thus:

$$x_t = x_o + v_{bo,x}t \text{ and } y_{bo} + v_{bo,y}t - \tfrac{1}{2}gt^2 = y_{to} + v_{to,y}t - \tfrac{1}{2}gt^2$$

We can simplify these somewhat: the "Y" equation simplifies to become $v_{bo,y}t = y_{to} + v_{to,y}t$ and for the x equation, we can suppose that x_o=0 to give $x_t = v_{bo,x}t$. We can find the ratio of these two to give:

$$\frac{v_{bo,y}t}{v_{bo,x}t} = \frac{v_{bo,y}}{v_{bo,x}} = \tan(\theta) = \frac{y_{to} + v_{to,y}t}{x_t}.$$

Clearly, there is only one case for which pointing at the target does insure hitting the target, namely at the very top of the target's trajectory. Thus, the easiest way to make sure you hit clay targets is to shoot at them right at the top of their trajectory by pointing at them.

Extra:

It is also possible to have an intersection in other cases. A method of solution is this: the time that it takes the bullet to travel to the target is:

$$t = \frac{x_t}{v_{bo,x}}$$

The angle to point at is then:

$$\tan(\theta) = \frac{y_{to} + v_{to,y}\left(\frac{x_t}{v_{bo,x}}\right)}{x_t} = \frac{y_{to} + x_t\left(\frac{v_{to,y}}{v_{bo,x}}\right)}{x_t} = \frac{y_{to}}{x_t} + \left(\frac{v_{to,y}}{v_{bo,x}}\right)$$

It would seem if the target is fairly low and the speed of the bullet is fairly high, a good approximation to the intersection is obtained by pointing at the target. In the context of trying to intercept an incoming missile, the problem is a lot more complicated than this due to the high speeds, the rotation of the earth, the curvature of the earth, and the non-linear acceleration of missiles. Also, the equations of motion would involve local non-uniform accelerations for both missiles along 3 coordinate directions. I suppose ultimately what this shows is that it is easier to hit a slow-moving target.

(4) The Earth has an average radius of R_e=6.37x10^6m. In order for an object to orbit the Earth, as a minimum it must cross a distance equal to the radius of the Earth in the time that it takes it to fall through the distance of the radius of the Earth (assuming the Earth is spherical). What is the minimum velocity which an object can have to orbit the Earth?

Solution: We require the following two equations: $R_e = V_{ox}t$ and $R_e = \frac{1}{2}gt^2$. Since the two are equal, and dropping the "ox" subscript, we have $Vt = \frac{1}{2}gt^2 \Rightarrow t = \frac{2V}{g}$. We use this in either equation to find (I am putting it into the first equation): $R_e = V(\frac{2V}{g}) = \frac{2V^2}{g} \Rightarrow v = \pm\sqrt{\frac{R_e g}{2}}$. Here, both solutions are physical. We'll use the + solution. We can then find the orbital velocity: v=5.59x10^3m/s. Note: this is not the escape velocity of the Earth which is about 11.2x10^3 m/s. This is a variation of the classical problem known as "Newton's Cannon." The orbital velocity is about 15500 mph which is 6.929x10^3 m/s . This is not bad agreement considering that I have solved for the case of the projectile just barely skimming along the surface of the Earth.

(5) It's fill-er-up time at the local dairy bar! Cowboy Ryan is seen busy behind the bar filling glasses of everyone's favorite malt beverage[1] and then sliding the glasses down to waiting customers. One of the customers missed catching the glass and it slid right off that bar and smashed onto the ground a distance of 4m from the base of the bar which was 1.5 m high. How fast was the glass moving when it left the bar?

[1] chocolate malt.

Solution: the equations of motion are: $x = x_o + v_{o,x}t$ and $y = y_o + v_{o,y}t - \frac{1}{2}gt^2$. We'll call the base of the bar x=0, y=0. Also, there is no initial velocity in the y direction. Thus, these two equations simplify to become: $x = v_{o,x}t$ and $0=y_o - \frac{1}{2}gt^2 \Rightarrow t = \pm\sqrt{\frac{2y_o}{g}}$. We use this time (the + solution is physical) in the x equation to obtain:

$$x = v_{ox}\sqrt{\frac{2y_o}{g}} \Rightarrow v_{o,x} = \frac{x}{\sqrt{\frac{2y_o}{g}}} = \frac{4}{\sqrt{\frac{2(1.5)}{9.8}}} = \frac{4}{0.553} = 7.23\,\frac{m}{s}.$$