	Physics 240: Worksheet 07	Name ________________

(1) A mass M_1 is on a frictionless table and is connected to a mass M_2 by a string which is hanging over the side of the table. Find the tension and the acceleration.

(2) Solve problem 1 where there is a frictional force f is present.

(3) Solve for the tension and acceleration of Atwood's machine.

(4) Suppose a block of wood of mass m is kicked along the floor with an initial velocity v. The block has a coefficient of kinetic friction of μ between the floor and the block. How far will the block slide till it stops. If the block has an initial velocity v=5 m/s and μ=0.3, provide a numerical answer together with correct units.

(5) Suppose that a force is applied to a mass which has the functional dependence shown below (b is constant, n>0). Find the resulting equations of motion.

(a) $\vec{F} = bt^n \hat{i}$

(b) $\vec{F} = bv^n \hat{i}$

(c) $\vec{F} = -|b| x \hat{i}$

(1) A mass M_1 is on a frictionless table and is connected to a mass M_2 by a string which is hanging over the side of the table. Find the tension and the acceleration.

Solution: Unbend the problem: **Use the free body diagrams as shown in class**.

$$\text{fb1:}\ \begin{array}{l} y: N - m_1 g = 0 \\ x: T = m_1 a \end{array} \qquad \text{fb2:}\ \begin{array}{l} m_2 g - T = m_2 a \\ \text{no other equation} \end{array}$$

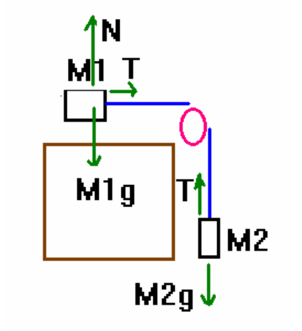
To solve:

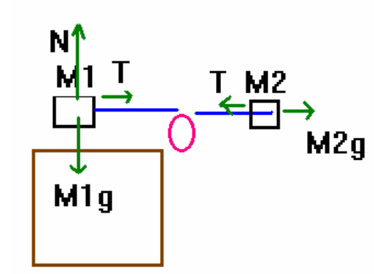
$N = m_1 g$ (no further use of this in this problem)

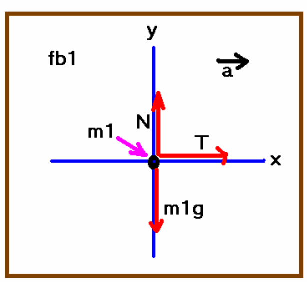
add $T=m_1 a$ and $m_2 g - T = m_2 a$:

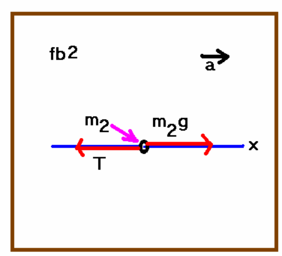
$$\begin{array}{r} T=m_1 a \\ + \quad m_2 g - T = m_2 a \\ \hline m_2 g = (m_1 + m_2) a \end{array}$$

$$m_2 g = (m_1 + m_2) a \Rightarrow a = \frac{m_2}{m_1+m_2} g \Rightarrow T = \frac{m_1 m_2}{m_1+m_2} g$$









(2) Solve problem 1 where there is a frictional force f is present.

Solution: Unbend the problem: **Use the free body diagrams as shown in class**.

$$\text{fb1}: \begin{array}{l} y: N - m_1 g = 0 \\ x: T - f = m_1 a \end{array} \quad \text{fb2}: \begin{array}{c} m_2 g - T = m_2 a \\ \text{no other equation} \end{array} \quad f = \mu N$$

$$N = m_1 g \;\Rightarrow f = \mu m_1 g \Rightarrow T - \mu m_1 g = m_1 a$$

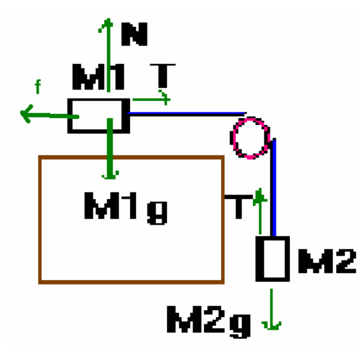
add $T - \mu m_1 g = m_1 a$ and $m_2 g - T = m_2 a$:

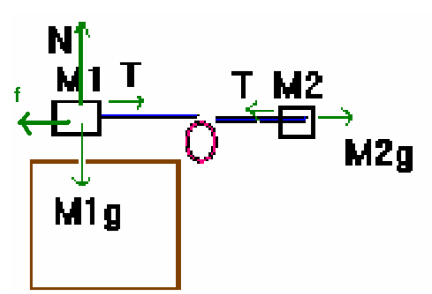
$$\begin{array}{r} T - \mu m_1 g = m_1 a \\ + \; m_2 g - T = m_2 a \\ \hline (m_2 - \mu m_1) g = (m_1 + m_2) a \end{array}$$

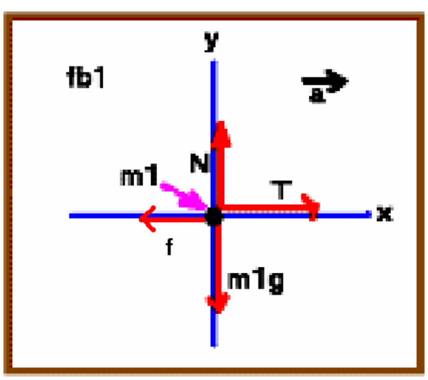
$$(m_2 - \mu m_1) g = (m_1 + m_2) a \Rightarrow a = \frac{(m_2 - \mu m_1)}{m_1 + m_2} g$$

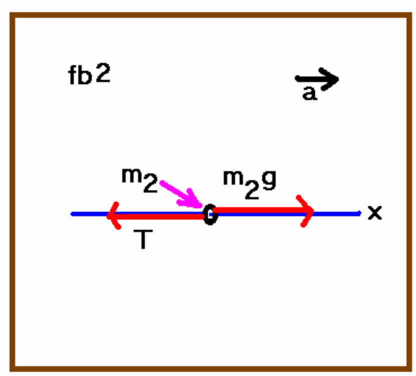
$$\Rightarrow T = \mu m_1 g + m_1 a = m_1 \left(\frac{(\mu m_1 + \mu m_2) + (m_2 - \mu m_1)}{m_1 + m_2} \right) g = (\mu + 1) m_1 \left(\frac{m_2}{m_1 + m_2} \right) g$$

Notice that if $\mu = 0$ (no friction) we have the previous problem.



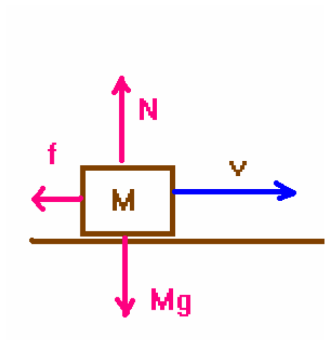




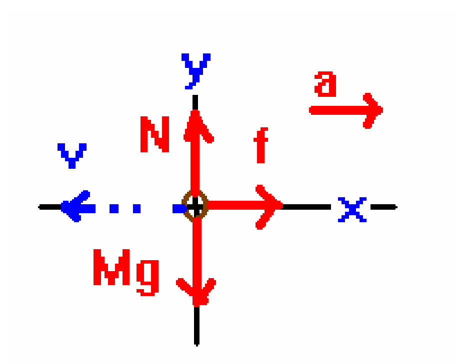


(3) Solve for the tension and acceleration of Atwood's machine.
Solution: An animated gif exists under the lab homepage which shows the steps needed to solve this problem (see lab 3).

(4) Suppose a block of wood of mass m is kicked along the floor with an initial velocity v. The block has a coefficient of kinetic friction of μ between the floor and the block. How far will the block slide till it stops. If the block has an initial velocity v=5 m/s and μ=0.3, provide a numerical answer together with correct units.



Here is a picture of the problem. The initial problem is that only external force which is not balanced here is the frictional force. It causes an acceleration in the opposite direction to the velocity. You are going to want to reverse the problem so that the acceleration is in the positive direction. The free body diagram for this is shown below. Remember: the acceleration is always in the direction of the net external force!
I have obeyed my rule here in that I have directed the acceleration in the +x direction which means that the initial velocity thus is in the –x direction.



Newton's laws give:
$\sum \vec{F} = m\vec{a} \Rightarrow \begin{matrix} N - Mg = 0 \\ f = Ma \end{matrix}$. Of course, we have the constitutive equation: $f = \mu N \Rightarrow f = \mu Mg$.
With the frictional force given above, we are now ready to solve for the acceleration:
$\mu Mg = Ma \Rightarrow a = \mu g$. How far does it go? This is given by one of the equations of motion:
$0 = v_i^2 + 2a(\Delta x) \Rightarrow \Delta x = -\frac{v_i^2}{2\mu g}$. The – sign is appropriate here. Now, putting numbers into this, we find $\Delta x = -\frac{25}{2(0.3)(9.8)} = 4.25\text{m}$.

(5) Suppose that a force is applied to a mass which has the functional dependence shown below (b is constant, n>0). Find the resulting equations of motion.
(a) $\vec{F} = bt^n\hat{i}$
(b) $\vec{F} = bv^n\hat{i}$
(c) $\vec{F} = -|b|x\,\hat{i}$

(a) $F = m\frac{dv}{dt} = bt^n \Rightarrow dv = \left(\frac{b}{m}\right)t^n dt \Rightarrow \int_{v_0}^{v} dv = \left(\frac{b}{m}\right)\int_{t_0}^{t} t^n dt \Rightarrow \Delta v = \left(\frac{b}{m}\right)\frac{t^{n+1} - t_0^{n+1}}{n+1}$

if at t=0, t_0=0 then we have the result: $v = v_0 + \frac{b}{m(n+1)}t^{n+1}$

$v = \frac{dx}{dt} \Rightarrow x = x_0 + v_0 t + \frac{b}{m(n+1)(n+2)}t^{n+2}$

(b) $F = bv^{n} = m\frac{dv}{dt} \Rightarrow \frac{dv}{v^{n}} = \left(\frac{m}{b}\right)dt \Rightarrow \int_{v_0}^{v} v^{-n}dv = \left(\frac{m}{b}\right)\int_{0}^{t} dt$

(i) if n=1, then $\ln\left(\frac{v}{v_0}\right) = \left(\frac{m}{b}\right)t \Rightarrow v = v_0 e^{\left(\frac{m}{b}\right)t}$

$$v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = \int_{0}^{t} v_0 e^{\left(\frac{m}{b}\right)t}dt \Rightarrow x = x_0 + v_0\left(\frac{m}{b}\right)\left[e^{\left(\frac{m}{b}\right)t} - 1\right]$$

(ii) if n<>1, then $v^{-n+1} = v_0^{-n+1} + \left(\frac{m}{b}\right)t \Rightarrow v = \left[v_0^{-n+1} + \left(\frac{m}{b}\right)t\right]^{n-1}$

$$v = \frac{dx}{dt} \Rightarrow \Delta x = \int_{0}^{t} vdt = \int_{0}^{t}\left[v_0^{-n+1} + \left(\frac{m}{b}\right)t\right]^{n-1}dt = \frac{\left[v_0^{-n+1} + \left(\frac{m}{b}\right)t\right]^{n}}{\left(\frac{m}{b}\right)n} - \frac{\left[v_0^{-n+1}\right]^{n}}{\left(\frac{m}{b}\right)n}$$

(c)
$$\frac{d^2x}{dt^2} = -\frac{b}{m}x^{n} \Rightarrow \frac{d^2x}{dt^2} - \left|\frac{b}{m}\right|x = 0 \Rightarrow x = A\cos(\omega t) + B\sin(\omega t)$$
$$v = \frac{dx}{dt} = -\omega A\sin(\omega t) - B\cos(\omega t) : \omega = \sqrt{\left|\frac{b}{m}\right|}$$

This is an example of a problem that we will soon study which is called the “simple harmonic oscillator”.