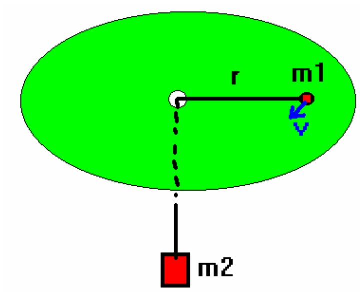
	Physics 240: Worksheet 12	Name ________________

(1) A particle of mass m is undergoing uniform circular motion with a radius r. Show that the acceleration is given by $a_c = \frac{v^2}{r}$, that the acceleration is directed towards the center of the circle and that the force producing this acceleration is given by $F = m\frac{v^2}{r}$.

(2) Suppose David of the Goliath type attached a 1kg stone to a piece of goat's gut which was 1 m long. How much force did David have to apply to the goat's gut in order to produce a tangential velocity of 10 m/s for the stone? What about 5 m/s?

(3) A mass m1 is lying on a frictionless table with a hole in the center at a distance r from the hole. A massless string is connected to m1 and passes through the hole to connect to a mass m2. What tangential velocity must m1 have in order to suspend m2?



(4) Suppose a penny is lying on a record. There is a coefficient of friction between the record and the penny given by μ, and the record is spinning with an angular velocity of ω. How far from the center of the record can the penny be before it starts sliding?

(5) The moon goes around the Earth once every 30 days. The mass of the moon is 7.4×10^{22} kg and the moon is about 1 light-second away from the Earth. How big is the centripetal force that keeps the moon in orbit?

(6) Suppose a spring of spring constant 10 N/m which is initially unstretched but of length 1 m is attached to a 1kg stone. If the stone has a velocity of 10 m/s, how long is the spring at this point?

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Solution: This derivation is presented as an animated gif. Please refer to the following URL for this derivation:

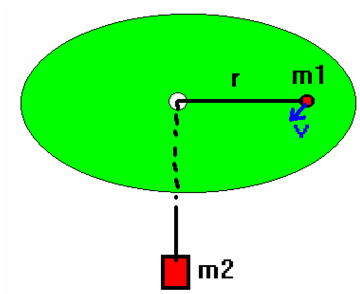
http://www.lyon.edu/webdata/users/shutton/animations/circle1.gif.

This is, however the non-calculus method. I refer you to the special notes regarding centripetal acceleration presented below this worksheet on our home page. Note that the vector form of the force is $\vec{F} = m\vec{a}_c = -m\frac{v^2}{r}\hat{r}$ where $\hat{r}$ is the unit vector in the radial direction.

(2) Suppose David of the Goliath type attached a 1kg stone to a piece of goat's gut which was 1 m long. How much force did David have to apply to the goat's gut in order to produce a tangential velocity of 10 m/s for the stone? What about 5 m/s?

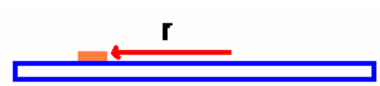
Solution: $F = m\frac{v^2}{r}$. Here, we have v=10 m/s, m=1 kg and r=1 m. Thus, $F = 1\left(\frac{10^2}{1}\right) = 100N$. What about if v=5 m/s? Then F=25 N … notice that the force is proportional to velocity ***squared***.

(3) A mass m1 is lying on a frictionless table with a hole in the center at a distance r from the hole. A massless string is connected to m1 and passes through the hole to connect to a mass m2. What tangential velocity must m1 have in order to suspend m2?

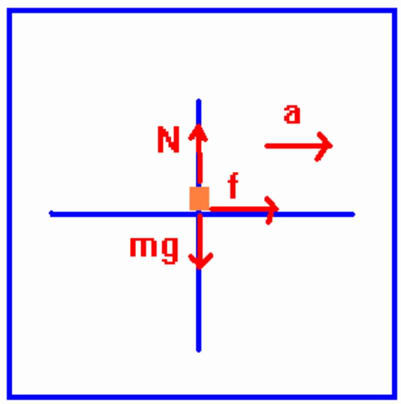


Solution: The centripetal force which keeps m1 moving in uniform circular motion is given by $F = m_1\frac{v^2}{r}$. This force must be equal to the weight of m_2 in order to suspend m_2 and, in fact, it is the weight of m_2 that is supplying this needed force. Thus $F=m_2g$. We solve these two equations for v: $m_1\frac{v^2}{r} = m_2 g \Rightarrow v^2 = rg\frac{m_2}{m_1} \Rightarrow v = \pm\sqrt{rg\frac{m_2}{m_1}}$. Either direction is perfectly valid as a solution to this problem.

(4) Suppose a penny is lying on a record. There is a coefficient of friction between the record and the penny given by μ, and the record is spinning with an angular velocity of ω. How far from the center of the record can the penny be before it starts sliding?



Let's sketch a free body diagram. I have oriented my picture so that **a** will appear to be in the +x direction. The force causing the centripetal acceleration here is the frictional force. To see which direction it is operating in, remember … we solve the problem with out friction to see which direction the penny would move. The frictional force is in the opposite direction. Thus, the free body diagram is:



We apply Newton's laws: $\sum \vec{F} = m\vec{a}$ so $N - mg = 0$ and $f = ma$ and we also have for the frictional force $f = \mu N$. We can put all this together: $f = \mu mg$ and thus: $\mu mg = ma = m\frac{v^2}{r}$. The mass will drop out of this to produce the following result: $\frac{v^2}{r} = \mu g$. The penny will slide when $v > \sqrt{\mu rg}$. Let's solve for the last radius at which it won't slide (i.e. solve the equality $v = \sqrt{\mu rg}$. Now for a point on the record, $v = \omega r$ so we can completely eliminate v from the problem: $\omega r = \sqrt{\mu rg}$. We want to put r alone on one side. This can be done as follows: $r = \frac{\mu g}{\omega^2}$. This is the absolute largest r that the penny can be at. If r gets any larger, the penny will slide.

(5) The moon goes around the Earth once every 30 days. The mass of the moon is $7.4\text{x}10^{22}$ kg and the moon is about 1 light-second away from the Earth. How big is the centripetal force that keeps the moon in orbit?

Solution: 1 light-second is $3.0\text{x}10^8$ m/s x 1 s=$3.0\text{x}10^8$ m. (a light-second is the distance that light travels in a second). The orbital period is $30 \text{ days x } \frac{86400\text{s}}{1\text{ day}} = 2.6\text{x}10^6\text{s}$. We need to find the angular velocity since $v = \omega r$, it will give us v.

The angular velocity is given by $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{"rad"}}{2.6\text{x}10^6\text{s}} = 2.4\text{x}10^{-6}\text{"rad"/s}$.

The tangential velocity is then $v = \omega r = 2.5\text{x}10^{-6}\text{x}3.0\text{x}10^8 = 750\text{m/s}$.

The force is then $F = m\frac{v^2}{r} = 7.4\text{x}10^{22}\text{x}\frac{(750)^2}{3.0\text{x}10^8} = 1.4\text{x}10^{20}\text{N}$.

This is the gravitational attraction between the moon and the Earth. I can prove this last statement to you by looking at Newton's law of Gravitation. We'll need some constants, however, to show this:

m_m=$7.4\text{x}10^{22}$ kg: m_e=$5.97\text{x}10^{24}$ kg: r_{em}=$3.0\text{x}10^8$ m: G=$6.67\text{x}10^{-11}$ $\text{m}^3/[\text{kg s}^2]$

The gravitational force is then:

$$\left|\vec{F}\right| = G\frac{m_e m_m}{r_{em}^2} = 6.67\text{x}10^{-11}\frac{\left[5.97\text{x}10^{24}\right]\left[7.4\text{x}10^{22}\right]}{\left[3.0\text{x}10^8\right]^2} = 3.3\text{x}10^{20}\text{N}$$

The difference between the two is in part due to the approximation that the distance is 1 light-second. In fact, the distance is closer to $3.85\text{x}10^8$ m. There are other factors here also.

(6) Suppose a spring of spring constant 10 N/m which is initially unstretched but of length 1 m is attached to a 1kg stone. If the stone has a velocity of 10 m/s, how long is the spring at this point?

The force exerted by the spring is equal to the centripetal force required to keep the stone in motion. The only complication here is that the force exerted by the spring is actually going to be given by:

$$F = k(R - R_0)$$

where R_0 is the initial unstretched radius. The solution then proceeds as follows:

$$F_{spring} = F_{centripetal} \Rightarrow k(R - R_0) = m\tfrac{v^2}{R} \Rightarrow R^2 + (-R_0)R - \left(\tfrac{m}{k}v^2\right) = 0$$

Using the values stated in the problem, we have:

$$R^2 - R - 10 = 0 \Rightarrow R = \tfrac{1 \pm \sqrt{1+40}}{2} = 3.7\,\text{m}$$

You can also see an approximate solution of 3.1 m is obtained by ignoring R_0.
Also you'll notice I have chosen the positive physical solution here.