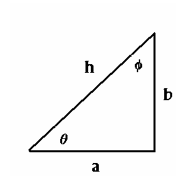
	Physics 240: Worksheet 01	Name ________________

(1) Which of the following equations are dimensionally correct? Note: the following quantities have the following dimensions:

$$F:\left[MLt^{-2}\right];m:[M];a:\left[Lt^{-2}\right];t:[t];x:[L];E:\left[ML^{2}t^{-2}\right];v:\left[Lt^{-1}\right];s:[L]$$

(a) F=ma
(b) $X=(1/2)at^3$
(c) E=(1/2)mv
(d) E=max
(e) $V=[Fs/m]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find sin(θ), cos(θ) and tan(θ), sin(φ), cos(φ) and tan(φ).



(3) A vector $\vec{A}$ is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:

(a) what is the magnitude of $\vec{A}$?
(b) what is the angle the vector makes with the x-axis?
(c) what is the angle the vector makes with the y-axis
(d) Express this vector using the "hat" notation.
(e) Express this vector using the "x-y" unit vector notation

(4) Suppose a vector $\vec{B}$ is given by $\vec{B}=3\hat{i}+2\hat{j}$. Find the following:

(a)What is $2\vec{B}$?
(b) What is $\vec{B}+\vec{A}$?
(c) What is $\vec{B}-\vec{A}$?
(d) What is $\vec{B}\bullet\vec{A}$? (dot product)
(e) What is the angle made with respect to the positive x-axis by $\vec{B}+\vec{A}$?

(5) Suppose a vector $\vec{C}$ is given by $\vec{C}=8\hat{i}-9\hat{j}$. A person walks along vector $\vec{A}$, then vector $\vec{B}$, followed by vector $\vec{C}$. At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

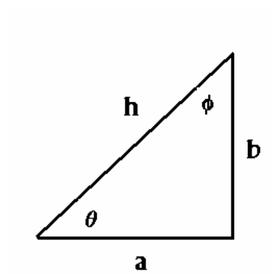
(1) Which of the following equations are dimensionally correct? Note: the following quantities have the following dimensions:

$$F:\left[MLt^{-2}\right];m:[M];a:\left[Lt^{-2}\right];t:[t];x:[L];E:\left[ML^2t^{-2}\right];v:\left[Lt^{-1}\right];s:[L]$$

(a) F=ma
(b) $X=(1/2)at^3$
(c) E=(1/2)mv
(d) E=max
(e) $V=[Fs/m]^{1/2}$

Solution:
(a) yes: F= $[M\,L\,t^{-2}]$ as dimensions. m= [M] and a= $[L\,t^{-2}]$.
So you can see that $[M\,L\,t^{-2}]= [M]\,[L\,t^{-2}]$.
(b) no: X=[L], a=$[Lt^{-2}]$ and t=[t] so $[L]\neq[Lt]$
(c) no: E=$[M][L^2t^{-2}]$, [M] and v=$[L\,t^{-1}]$ so $[ML^2t^{-2}]\neq[MLt^{-1}]$
(d) yes: E=$[M][L^2t^{-2}]$, a= $[L\,t^{-2}]$, x=[L] so $[ML^2t^{-2}]=[ML^2t^{-2}]$
(e) yes: v=$[L\,t^{-1}]$, F= $[M\,L\,t^{-2}]$, s=[L], m=[M] so $[L\,t^{-1}]=[L^2t^{-2}]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find sin(θ), cos(θ) and tan(θ), sin(φ), cos(φ) and tan(φ).



Solution:
(a) $h=\sqrt{a^2+b^2}$
(b) $\sin(\theta)=\frac{b}{h}\quad \cos(\theta)=\frac{a}{h}\quad \tan(\theta)=\frac{b}{a}$
(c) $\sin(\varphi)=\frac{a}{h}\quad \cos(\varphi)=\frac{b}{h}\quad \tan(\varphi)=\frac{a}{b}$

(3) A vector $\vec{A}$ is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:

(a) what is the magnitude of $\vec{A}$?
(b) what is the angle the vector makes with the x-axis?
(c) what is the angle the vector makes with the y-axis
(d) Express this vector using the "hat" notation.
(e) Express this vector using the "x-y" unit vector notation

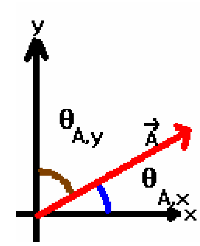
Solution:

(a) $\left|\vec{A}\right|=\sqrt{\vec{A}\bullet\vec{A}}=\left[25+16\right]^{\frac{1}{2}}=\sqrt{41}=6.403$ |A|=[A•A]1/2=[52+42]1/2=6.403

(b) $\cos(\theta_{A,x})=\frac{\vec{A}\bullet\hat{x}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\bullet(\hat{x}+0\hat{y})}{\sqrt{5^2+4^2}}=\frac{5}{6.403}=0.7809\quad \text{So},\theta_{A,x}=\cos^{-1}(0.7809)=38.66^0$

(c) $\cos(\theta_{A,y})=\frac{\vec{A}\bullet\hat{y}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\bullet(0\hat{x}+\hat{y})}{\sqrt{5^2+4^2}}=\frac{4}{6.403}=0.6247\quad \text{So},\theta_{A,y}=\cos^{-1}(0.6247)=51.34^0$

My unusual notation for these angles is for clarity: these are angles with respect to the particular axis of interest. Also, note that these 2 angles add up to be 90 degrees.



(d) $\vec{A}=5\hat{i}+4\hat{j}$
(e) $\vec{A}=5\hat{x}+4\hat{y}$

(4) Suppose a vector $\vec{B}$ is given by $\vec{B} = 3\hat{i} + 2\hat{j}$. Find the following:

(a)What is $2\vec{B}$?

(b) What is $\vec{B} + \vec{A}$?

(c) What is $\vec{B} - \vec{A}$?

(d) What is $\vec{B} \bullet \vec{A}$? (dot product)

(e) What is the angle made with respect to the positive x-axis by $\vec{B} + \vec{A}$?

Solution:

(a) $2\vec{B} = (2\times3)\hat{i} + (2\times2)\hat{j} = 6\hat{i} + 4\hat{j}$

(b) $\vec{B} + \vec{A} = \left[3\hat{i} + 2\hat{j}\right] + \left[5\hat{i} + 4\hat{j}\right] = (3+5)\hat{i} + (2+4)\hat{j} = 8\hat{i} + 6\hat{j}$

(c) $\vec{B} - \vec{A} = \left[3\hat{i} + 2\hat{j}\right] - \left[5\hat{i} + 4\hat{j}\right] = (3-5)\hat{i} + (2-4)\hat{j} = -2\hat{i} - 2\hat{j}$

(d) $\vec{B} \bullet \vec{A} = \left[3\hat{i} + 2\hat{j}\right] \bullet \left[5\hat{i} + 4\hat{j}\right] = (3\times5) + (2\times4) = 15 + 8 = 23$

(e) $\cos\left(\theta_{\vec{B}+\vec{A},\hat{x}}\right) = \frac{\left[\vec{B}+\vec{A}\right]\bullet\hat{i}}{\left|\vec{B}+\vec{A}\right|} = \frac{\left[8\hat{i}+6\hat{j}\right]\bullet\hat{i}}{\sqrt{8^2+6^2}} = \frac{8}{\sqrt{100}} = \frac{8}{10} = 0.8 \Rightarrow \theta = 36.87^o$

Note that the power of the method in (e) gives you a manner to obtain the angle with respect to any unit vector. You can define a unit vector in an arbitrary direction easily as:

$$\hat{B} \equiv \frac{\vec{B}}{\left|\vec{B}\right|}.$$

For example, for this particular vector, we have: $\hat{B} = \frac{3\hat{i}+2\hat{j}}{\sqrt{3^2+2^2}} = \frac{3}{\sqrt{13}}\hat{i} + \frac{2}{\sqrt{13}}\hat{j} = 0.832\hat{i} + 0.555\hat{j}$.

To find the angle between the vectors A and B, you would then take, for example:

$$\cos\left(\theta_{\vec{A},\vec{B}}\right) = \frac{\vec{A}\bullet\hat{B}}{\left|\vec{A}\right|} = \frac{\left[5\hat{i}+4\hat{j}\right]\bullet\left[0.832\hat{i}+0.555\hat{j}\right]}{\sqrt{5^2+4^2}} = \frac{4.16+2.22}{\sqrt{41}} = \frac{6.38}{\sqrt{41}} = 0.996 \Rightarrow \theta_{\vec{A},\vec{B}} = 4.87^0$$

You could, of course, use the law of cosines which states:

$$\cos\left(\theta_{\vec{A},\vec{B}}\right) = -\frac{\left|\vec{B}-\vec{A}\right|^2 - \left|\vec{A}\right|^2 - \left|\vec{B}\right|^2}{2\left|\vec{A}\right|\left|\vec{B}\right|} = -\frac{\left[\vec{A}\bullet\vec{A}+\vec{B}\bullet\vec{B}-2\vec{A}\bullet\vec{B}\right]-\vec{A}\bullet\vec{A}-\vec{B}\bullet\vec{B}}{2\left|\vec{A}\right|\left|\vec{B}\right|} = \frac{\vec{A}\bullet\vec{B}}{\left|\vec{A}\right|\left|\vec{B}\right|} \equiv \hat{A}\bullet\hat{B} = \frac{\vec{A}\bullet\hat{B}}{\left|\vec{A}\right|}$$

where I have used the fact that the vector pointing from $\vec{A}$ to $\vec{B}$ is given by $\vec{B} - \vec{A}$.

(5) Suppose a vector $\vec{C}$ is given by $\vec{C} = 8\hat{i} - 9\hat{j}$. A person walks along vector $\vec{A}$, then vector $\vec{B}$, followed by vector $\vec{C}$. At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

Solution:

$\vec{D} = \vec{A} + \vec{B} + \vec{C} = \left[5\hat{i} + 4\hat{j}\right] + \left[3\hat{i} + 2\hat{j}\right] + \left[8\hat{i} - 9\hat{j}\right] = (5+3+8)\hat{i} + (4+2-9)\hat{j} = 16\hat{i} + (-3)\hat{j}$

$\left|\vec{D}\right| = \sqrt{\vec{D}\bullet\vec{D}} = \sqrt{16^2 + 3^2} = \sqrt{256+9} = \sqrt{265} = 16.279$

You can easily verify that $\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{0}$.

The vector symbol over 0 is necessary because the result of adding two vectors must produce a vector.