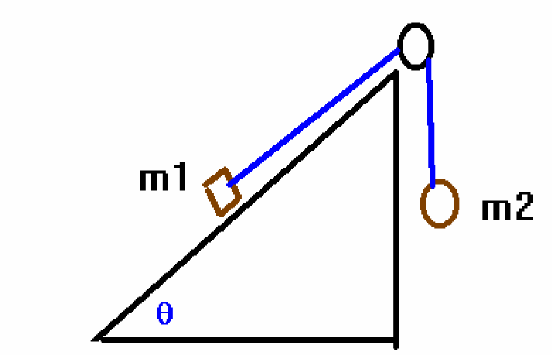
	Physics 240: worksheet 08	Name ________________

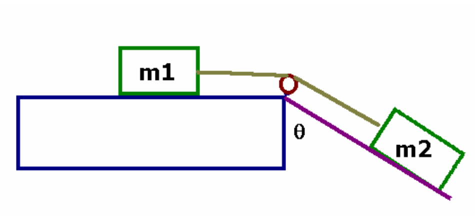
(1) A mass m is on an inclined plane which is inclined at an angle θ. If there is no friction in the system, (a) find the acceleration of the mass. If the mass is started from rest at an initial height of d above the bottom of the plane, answer the following: (b) How fast is it moving at the bottom and (c) how long does it take to slide down the plane.

(2) A mass m is on an inclined plane which is inclined at an angle θ. If the coefficient of friction between the inclined plane and the mass is μ, at what angle should the plane be tilted in order for the block to slide with a constant velocity after being given an initial small push?

(3) Find the acceleration of the system shown when $m_2 > m_1$. The system is frictionless.



(4) A mass m_1 is resting on a table and is connected by a string to a mass m_2 which is on an inclined plane as shown. The coefficient of friction is μ between m_1 and the table. If the system is given an initial velocity in the direction of m_2 to overcome friction, find the tension and the acceleration of the system.

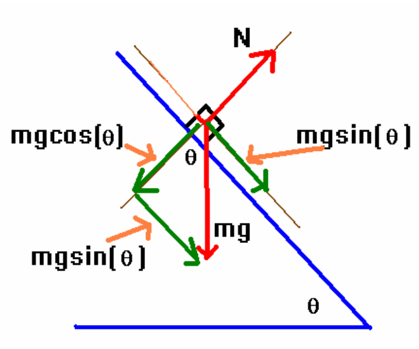


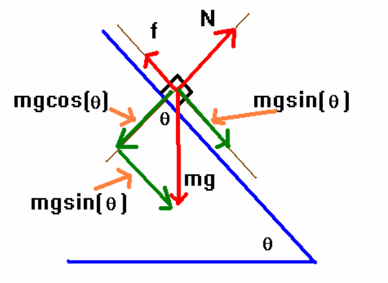
(5) Analyze Atwood's machine to obtain the tension and the acceleration of the Atwood's machine.

	Physics 240: worksheet 08	Name ________________

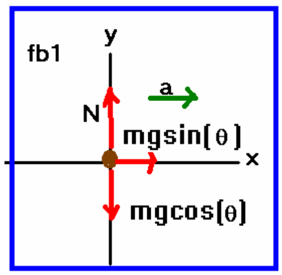
(1) A mass m is on an inclined plane which is inclined at an angle θ. If there is no friction in the system, (a) find the acceleration of the mass. If the mass is started from rest at an initial height of d above the bottom of the plane, answer the following: (b) How fast is it moving at the bottom and (c) how long does it take to slide down the plane.

Important note: For the inclined plane, remember this constructions:





Looking at the above construction, you will want to construct a free body diagram. You can either rotate your mind or you can rotate your head. It's your choice.



Apply Newton's laws: $\sum \vec{F} = m\vec{a} \Rightarrow \begin{array}{l} y: N - mg\cos(\theta) = 0 \Rightarrow N = mg\cos(\theta) \\ x: mg\sin(\theta) = ma \Rightarrow a = g\sin(\theta) \end{array}$

If the mass started at a distance d above the bottom of the plane, when it reached the bottom of the plane, it traveled through a distance related to d by:

$$d = (\Delta x)\sin(\theta) \Rightarrow \Delta x = \frac{d}{\sin(\theta)}$$

We can find how fast it is moving at the bottom of the plane from the third equation of motion:

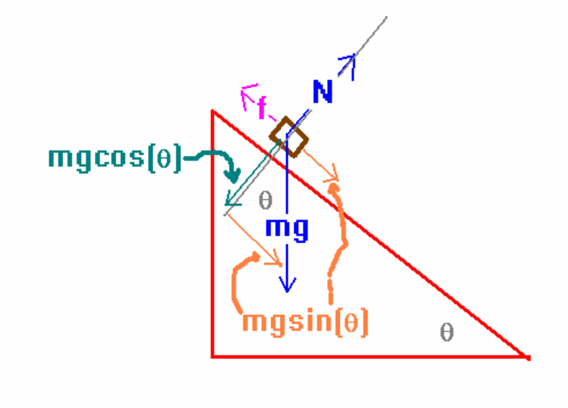
$$v^2 = v_{0,x}^2 + 2a_x(\Delta x) \Rightarrow v = \pm\sqrt{2a_x(\Delta x)} = \pm\sqrt{2(g\sin(\theta))\left(\frac{d}{\sin(\theta)}\right)} = \pm\sqrt{2gd}$$

The physical solution is the positive solution. This is exactly the same result that would have happened if the object had fallen straight down (true, under frictionless conditions). We can find out how long it takes for this to happen from the second equation of motion:

$$v_x = v_{0,x} + a_x t \Rightarrow t = \frac{v_x - v_{0,x}}{a_x} = \frac{\sqrt{2gd}}{a_x} = \frac{\sqrt{2gd}}{g\sin(\theta)}$$

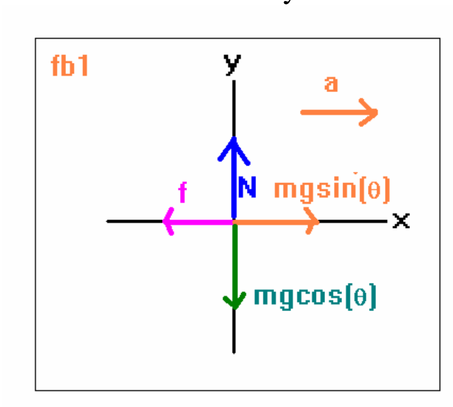
This correctly represents that at zero inclination, it never gets to the bottom.

(2) A mass m is on an inclined plane which is inclined at an angle θ. If the coefficient of friction between the inclined plane and the mass is μ, at what angle should the plane be tilted in order for the block to slide with a constant velocity after being given an initial small push?



Then, assuming the mass has started moving, find the acceleration of the system.

Solution: Sketch the system as shown. Then, draw the "rotated" free body diagram.



Let's solve this as if there were an acceleration:

$\sum \vec{F} = m\vec{a}$. This gives:

$y: N - mg\cos(\theta) = 0 \Rightarrow N = mg\cos(\theta)$

$x: mg\sin(\theta) - f = ma_x$

And we have the constitutive relationship:

$f = \mu N$. We then have from the y direction the following result: $f = \mu mg\cos(\theta)$

This then gives for the x-direction the result:

$mg\sin(\theta) - \mu mg\cos(\theta) = ma_x$. We can simplify this a bit to become:

$a_x = g\left[\sin(\theta) - \mu\cos(\theta)\right]$. Now, we want it to slide constantly down the plane. This means a=0. Thus, for this to happen:

$$\sin(\theta_{a=0}) - \mu\cos(\theta_{a=0}) = 0 \Rightarrow \mu = \tan(\theta_{a=0}) .$$

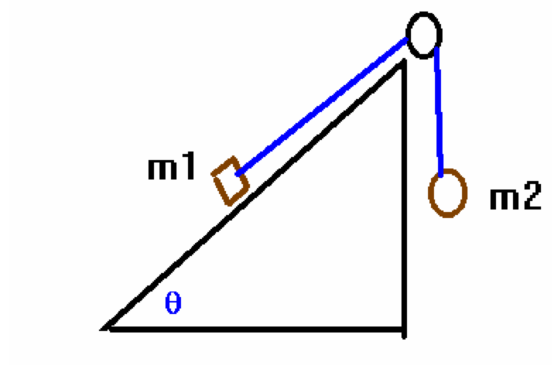
You can also solve this to answer the question of what the acceleration is after the system has started moving. Looking at the x-equation, we have:

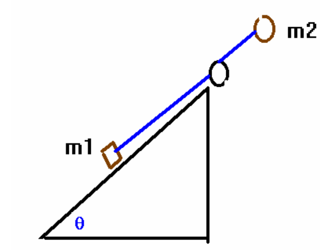
$$mg\sin(\theta) - \mu mg\cos(\theta) = ma_x \Rightarrow a_x = g\left[\sin(\theta) - \mu\cos(\theta)\right]$$

This applies at any angle. It happens to be zero when $\tan(\theta_{a=0}) = \mu_k$. If you go to an angle less than this, notice that the acceleration is in the direction of the frictional force, although the initial velocity is still down the plane.

If you wanted to answer questions about speed and velocity as in the second problem, you will need to follow through as you did in the second problem. Also notice, that when there is no friction $\mu = 0$ gives us exactly the same result as in the first problem.

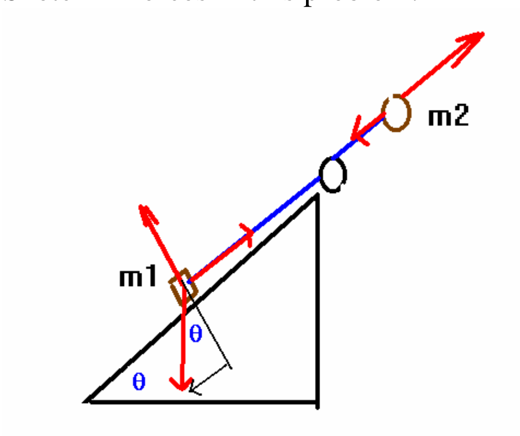
(3) Find the acceleration of the system shown when $m_2 > m_1$. The system is frictionless.



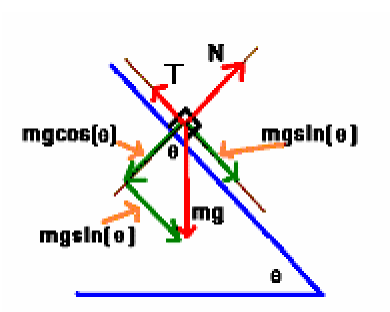


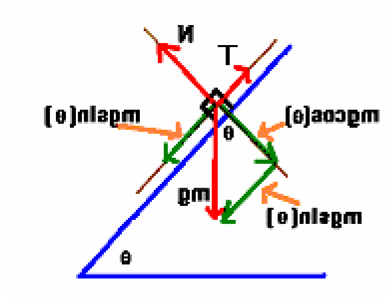
Unbend the problem as shown above.

Sketch in forces in this problem:

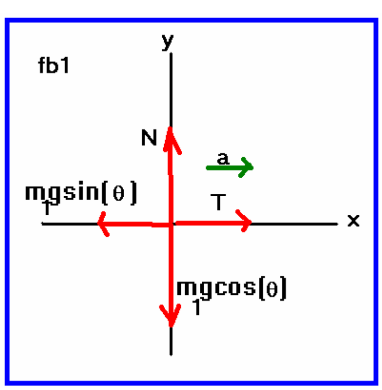


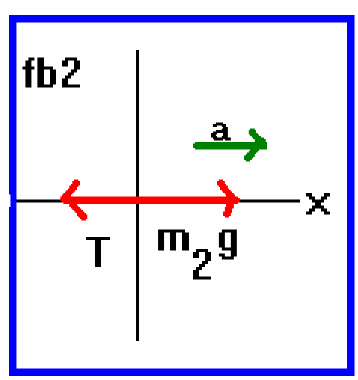
Observe the construction below and it's mirror image:





Draw 2 free body diagrams





Apply Newton's laws:

$$\sum \vec{F} = m\vec{a} \Rightarrow \text{fb1}: \begin{array}{c} y: N - m_1 g\cos(\theta) = 0 \Rightarrow N = m_1 g\cos(\theta) \\ x: T - m_1 g\sin(\theta) = m_1 a_x \end{array}$$

$$\Rightarrow \text{fb2}: x: m_2 g - T = m_2 a_x$$

$$\begin{array}{c} T - m_1 g\sin(\theta) = m_1 a_x \\ +\ m_2 g - T = m_2 a_x \\ \hline g\left(m_2 - m_1\sin(\theta)\right) = (m_1 + m_2)a_x \end{array}$$

$$\Rightarrow a_x = g\frac{\left(m_2 - m_1\sin(\theta)\right)}{(m_1 + m_2)}$$

$$\Rightarrow T = m_1 g\sin(\theta) + m_1 g\frac{\left(m_2 - m_1\sin(\theta)\right)}{(m_1 + m_2)}$$

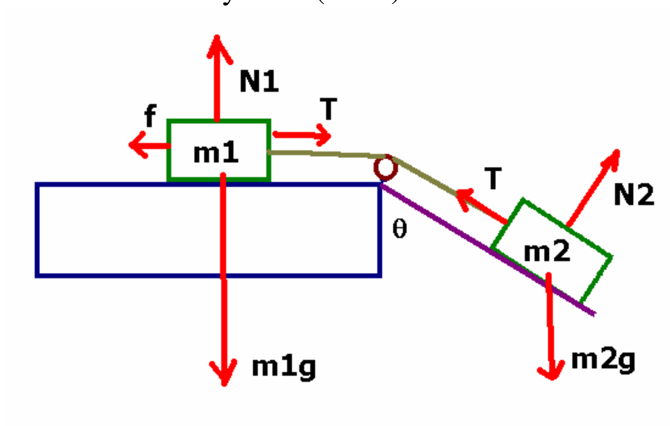
$$= g\left[\frac{(m_1 + m_2)m_1\sin(\theta) + m_1 m_2 - m_1^2\sin(\theta)}{(m_1 + m_2)}\right]$$

$$= g\left[\frac{m_1^2\sin(\theta) + m_1 m_2\sin(\theta) + m_1 m_2 - m_1^2\sin(\theta)}{(m_1 + m_2)}\right]$$

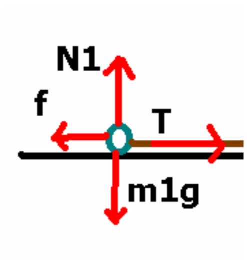
$$= g\left[\frac{m_1 m_2\left(1 + \sin(\theta)\right)}{(m_1 + m_2)}\right]$$

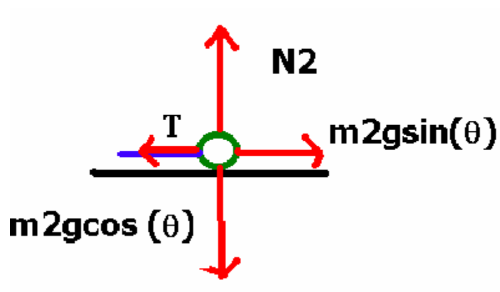
$$T = g\left[\frac{1}{\frac{1}{m_1} + \frac{1}{m_2}}\right]\left(1 + \sin(\theta)\right)$$

(4) A mass m_1 is resting on a table and is connected by a string to a mass m_2 which is on an inclined plane as shown. The coefficient of friction is μ between m_1 and the table. If the system is given an initial velocity in the direction of m_2 to overcome friction, find the tension and the acceleration of the system. (fixed)



Unbend the problem as shown:





Newton's laws say:

$$\text{fb1}: \begin{array}{l} T - f = m_1 a \\ N_1 - m_1 g = 0 \end{array} \qquad \text{fb2}: \begin{array}{l} m_2 g \sin(\theta) - T = m_2 a \\ N_2 - m_2 g \cos(\theta) = 0 \end{array}$$

$$f = \mu N_1$$

We can solve these equations now.

$$\left[T - \mu m_1 g = m_1 a\right]$$

$$+\left[m_2 g \sin(\theta) - T = m_2 a\right]$$

$$\Rightarrow m_2 g \sin(\theta) - \mu m_1 g = (m_1 + m_2) a \Rightarrow a = g \frac{m_2 \sin(\theta) - \mu m_1}{m_1 + m_2}$$

$$\Rightarrow T - \mu m_1 g = g(m_1 + m_2) \frac{m_2 \sin(\theta) - \mu m_1}{m_1 + m_2}$$

$$\Rightarrow T = g(m_1 + m_2) \frac{m_2 \sin(\theta) - \mu m_1}{m_1 + m_2} + \mu m_1 g$$

(5) Analyze Atwood's machine to obtain the tension and the acceleration of the system. This system is presented as an animated gif in the class home page and also in the lab home page. Please look at the analysis.
http://www.lyon.edu/webdata/users/shutton/animations/atwood2.gif