	Physics 240: Worksheet 14	Name ________________

You should refer to the notes about conservation of momentum before doing this worksheet.

(1) A mass m_1 is moving with an initial velocity $\vec{v}_1$. The mass inelastically collides with a second mass m_2 which is not moving. With what final velocity does the pair have after moving off?

(2) A mass m_1 collides elastically with a mass m_2 which is initially at rest. If $m_2 >> m_1$, with what velocity does m1 and m2 move off with?

(3) A mass m1 collides elastically with a mass m2 which is initially at rest. If m1=m2, with what velocity does m1 and m2 move off with?

(4) A mass m_1 collides elastically with a mass m_2 which is initially at rest If $m_1 >> m_2$, with what velocity does m_1 and m_2 move off with?

(5) It's a snowy day at the parking lot and two very similar cars (with the same mass) have a collision. The second car was not moving and had its breaks on and was knocked for 10 m at an angle (negative) of 35^0 from an arbitrarily placed bisector. The driver of the first car immediately applied the brakes and was diverted for 20m before stopping at an angle (positive) of 45^0 from the bisector. If the coefficient of friction between the snow and the cars is μ=0.2, how fast was the first car moving before the collision?

(1) A mass m_1 is moving with an initial velocity $\vec{v}_1$. The mass inelastically collides with a second mass m_2 which is not moving. With what final velocity does the pair have after moving off?
Solution: Although momentum is conserved here, kinetic energy is not since the collision is inelastic. Thus,

$$\Delta P = 0 \Rightarrow m_1 v_{before} = (m_1 + m_2) v_{after} \Rightarrow v_{after} = \frac{m_1}{m_1+m_2} v_{before}$$

As a numerical example, you might consider m_1=3 kg, m_2=5 kg and v_{before}=10 m/s. Then:

$$v_{after} = \frac{m_1}{m_1+m_2} v_{before} \Rightarrow v_{after} = \frac{3}{3+5} 10 = \frac{30}{8} = 3.75 m/s$$

(2) A mass m_1 collides elastically with a mass m_2 which is initially at rest. If $m_2 >> m_1$, with what velocity does m_1 and m_2 move off with?
Solution: This is kind-of like bouncing a basket ball off of a wall. The wall doesn't move much and the basket ball comes back at you. Let's see how this comes out of our equations. We had,

$$v_{1,f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1,i} \text{ and } v_{2,f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1,i}$$

To get the approximation here, we divide the top and the bottom of each by m_2:

$$v_{1,f} = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}\right) v_{1,i} \text{ and } v_{2,f} = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right) v_{1,i}$$

Now let the ratio become zero to find:

$$v_{1,f} \approx -v_{1,i} \text{ and } v_{2,f} \approx 0$$

Thus, the wall stays put and the basket ball comes back at the thrower.

(3) A mass m_1 collides elastically with a mass m_2 which is initially at rest. If $m_1=m_2$, with what velocity does m_1 and m_2 move off with?

Solution:
This is the case of two identical balls hitting each other. What is observed is that the energy is completely transferred from the first ball to the second ball. This needs to come out of the equations also. Let's see if it does. Our equations are:

$$v_{1,f} = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1,i} \text{ and } v_{2,f} = \left(\frac{2m_1}{m_1+m_2}\right)v_{1,i}$$

Now, let's let each of the masses be identical and equal to m. Thus,

$$v_{1,f} = \left(\frac{m-m}{m+m}\right)v_{1,i} \text{ and } v_{2,f} = \left(\frac{2m}{m+m}\right)v_{1,i}$$

We can then say that the result is:

$$v_{1,f} \approx 0 \text{ and } v_{2,f} \approx v_{1,i}\,.$$

In other words, the first ball stops and the second ball moves off with the speed of the first ball.

(4) A mass m_1 collides elastically with a mass m_2 which is initially at rest If $m_1>>m_2$, with what velocity does m_1 and m_2 move off with?
Solution: This is kind-of like bouncing a big bowling ball off of a ping-pong ball. We had for totally elastic collisions the following results:

$$v_{1,f} = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1,i} \text{ and } v_{2,f} = \left(\frac{2m_1}{m_1+m_2}\right)v_{1,i}$$

To get the approximation, divide each of these on top and bottom by m_1:

$$v_{1,f} = \left(\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)v_{1,i} \text{ and } v_{2,f} = \left(\frac{2}{1+\frac{m_2}{m_1}}\right)v_{1,i}$$

Now, let the ratios become about equal to zero. This gives the surprising results:

$$v_{1,f} \approx v_{1,i} \text{ and } v_{2,f} \approx 2v_{1,i}$$

(5) It's a snowy day at the parking lot and two very similar cars (with the same mass) have a collision. The second car was not moving and had its breaks on and was knocked for 10 m at an angle (negative) of 35^0 from an arbitrarily placed bisector. The driver of the first car immediately applied the brakes and was diverted for 20m before stopping at an angle (positive) of 45^0 from the bisector. If the coefficient of friction between the snow and the cars is μ=0.2, how fast was the first car moving before the collision?
Solution:
We can find out how fast the second car was traveling immediately after the collision:
$\Delta K_{NC} = \Delta K_c \Rightarrow -\mu mgx = -\frac{1}{2}mv^2 \Rightarrow v = \sqrt{2\mu gx}$ where I've assume a positive velocity. We can now answer questions about the initial velocity of the first car by finding the components of the final velocities. **We only really know for sure, in this collision, that the momentum was conserved at the moment of impact.**
For this situation, the velocity of the second car right after the collision was 6.26 m/s.
How? $v = \sqrt{2\mu gx} = \sqrt{2(.2)(9.8)(10)} = 6.261\frac{m}{s}$

The momentum components are: $$\begin{aligned} P_{x,2} &= m_2 v_{x,2} = m_2(6.26)\cos(-35) = 5.13m_2 \\ P_{y,2} &= m_2 v_{y,2} = m_2(6.26)\sin(-35) = -3.59m_2 \end{aligned}.$$

We can, in the same way, find out how fast the first car was moving after the collision, namely at 8.85 m/s.
How? $v = \sqrt{2\mu gx} = \sqrt{2(.2)(9.8)(20)} = 8.854\frac{m}{s}$
The components of momentum for the first car are:

$$P_{x,1} = m_1 v_{x,1} = m_1(8.85)\cos(45) = 6.25m_1$$
$$P_{y,1} = m_1 v_{y,1} = m_1(8.85)\sin(45) = 6.25m_1$$

Since momentum is conserved, we then have:

$$P_{x,1,before} = P_{x,1,after} + P_{x,2,after}$$
$$P_{y,1,before} = P_{y,1,after} + P_{y,2,after}$$

Since the masses are the same, we thus have:

$$v_{x,1,before} = 6.25 + 5.13 = 11.38m/s$$
$$v_{y,1,before} = 6.26 - 3.59 = 2.66m/s$$

We can now answer a question about how fast the first car was moving:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{11.38^2 + 2.66^2} = 11.69m/s.$$

If you decide to become an accident investigator, however, before you go to court make sure you've checked the kinetic energy of the situation or you'll have an unpleasant time in court.

One of the morals to this story is that not all collisions are elastic or inelastic and know your physics before you become an expert witness.