

Instructions: You have a total of 55 minutes to complete this test.

Answer each of the following questions completely.

Time Start _____ Time finish _____ Pledged _____

You must supply all details that led to your answer.

You must provide correct SI units where required.

Do not discuss any aspect of this test with anyone until I return the test.

Although you may use additional sheets of paper which should be turned in with your test, please write (neatly) your answers on the pages where the problems are presented.

Constants: $e^- = -1.602 \times 10^{-19} \text{ C}$, $k = 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$, $\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$

(1) A capacitor has a geometrical capacitance of $C_{\text{geo}} = 10 \mu\text{f}$. A material having a dielectric constant $\kappa = 3.5$ is inserted completely inside the capacitor.

(a) What is the new capacitance of the system?

(b) Suppose that the capacitance was equal to $5 \mu\text{f}$. If two of these capacitors are connected in series. Calculate the equivalent capacitance.

(c) Suppose that the capacitance was equal to $5 \mu\text{f}$. If two of these capacitors are connected in parallel. Calculate the equivalent capacitance.

A rod had a cross sectional area $A = 1 \times 10^{-4} \text{ m}^2$ and it is 1.5 m long. The material which the rod is made of has a resistivity given by $\rho_R = 1 \times 10^{-6} \Omega\text{m}$.

(e) Calculate the resistance of the rod.

(f) Suppose the resistance of the rod was 0.5Ω . If two of these rods are connected in series. Calculate the equivalent resistance.

(g) Suppose the resistance of the rod was 0.5Ω . If two of these rods are connected in parallel. Calculate the equivalent resistance.

Note: on problem 2 you must show complete details, using sketches. Simply putting down answers will not obtain much credit here.

(2) A parallel plate capacitor has plates of area A and separation d . On the plate located at the origin, a surface charge density $+\sigma$ exists while on the plate located at d a surface charge density $-\sigma$ exists.

Answer the following assuming that this can be regarded as an ideal capacitor.

(a) Starting from Gauss's Law, show (showing complete details) how to calculate the vector electric field within the capacitor in terms of σ, ϵ_0 and \hat{x} .

(b) Calculate the **magnitude** of the potential difference between the two plates in terms of σ, ϵ_0 and d .

(c) Calculate the capacitance of the capacitor in terms of ϵ_0, A , and d .

(d) How much energy is stored in this capacitor if the capacitance is 1.5×10^{-8} f when the potential difference is 1000 V? Be sure to include correct SI units with your answer.

(e) Now suppose that in some region of space the electric potential is given by:

$V = ax^3 + bxy + cy^3$ where a, b and c are constants. What is the vector electric field in this region of space?

(3) Consider the following charge configuration: consisting of three charges:

$$[1: -3\mu\text{C} : -2\text{m}, 0, 0]$$

$$[\text{Charge name: } Q : x, y, z] = [2: +3\mu\text{C} : 2\text{m}, 0, 0]$$

$$[3: 5\mu\text{C} : 0, 0, 0]$$

In each of the following problems, you should provide a symbolic answer followed by a numerical answer with correct SI units and Cartesian unit vectors where appropriate.

(a) Find the vector electric field at points along the axis specified by: $\vec{r}_p = 0\hat{x} + y_p\hat{y}$ (you may assume y_p is positive here and $y_p > 0$). Evaluate your answer for the case $y_p = 1.0\text{m}$.

(b) Find the electric potential at points along the axis specified by: $\vec{r}_p = 0\hat{x} + y_p\hat{y}$ (you may assume y_p is positive here and $y_p > 0$). Evaluate your answer for the case $y_p = 1.0\text{m}$.

(4) A sphere of charge of radius a has a volume charge distribution $\rho = \rho_0 \left[\frac{r}{a} \right]^2$. In addition to this, at the very center of the sphere is a point charge Q . **Note:** the radius of the sphere (a) also appears in the charge distribution as indicated.

Note: you must show complete details here, including sketches, etc. You may use the unit vector \hat{r} to represent the radial direction..

(a) Find the **vector** electric field inside the sphere.

(b) Find the **vector** electric field outside the sphere.

(c) Show that at the surface of the sphere the inside solution agrees with the outside solution.

(d) Find the potential at a point r_p outside the sphere.

(e) Suppose that $Q = -1\mu\text{C}$; $\rho_0 = \frac{10.0\mu\text{C}}{\text{m}^3}$; $a = 0.1\text{m}$. Provide a numerical solution for the electric field at the surface of the sphere. Be sure to include correct SI units with your answer.