Disk of charge along symmetry axis

Let the disk have a surface charge σ and be located in the x-y plane at z=0.

The point of field calculation is directly above the center of the disk.

$$\vec{r}_i = x_i\hat{x} + y_i\hat{y} + z_i\hat{z} : \vec{r}_p = 0\hat{x} + 0\hat{y} + z_p\hat{z} : \vec{r}_{ip} = -x_i\hat{x} - y_i\hat{y} + \left(z_p - z_i\right)\hat{z}$$

$$\vec{E} = \iint_q \frac{k[\sigma]}{\left[x_i^2+y_i^2+\left(z_p-z_i\right)^2\right]}\left[\frac{-x_i\hat{x}-y_i\hat{y}+\left(z_p-z_i\right)\hat{z}}{\sqrt{x_i^2+y_i^2+\left(z_p-z_i\right)^2}}\right]dx_i dy_i = k\sigma\iint_q \frac{-x_i\hat{x}-y_i\hat{y}+\left(z_p-z_i\right)\hat{z}}{\left[x_i^2+y_i^2+\left(z_p-z_i\right)^2\right]^{\frac{3}{2}}}dx_i dy_i$$

Convert to polar coordinates and let disk be at the origin.

$$\vec{E} = k\sigma\int_{s=0}^{a}\int_{\varphi=0}^{2\pi}\frac{-s\cos\varphi\hat{x}-s\sin\varphi\hat{y}+z_p\hat{z}}{\left[s^2+z_p^2\right]^{\frac{3}{2}}}s\,ds\,d\varphi$$

The x and y components integrate to give zero because the integrals over the sin and cosine will integrate to give zero.

The only remaining integral is then:

$$\vec{E} = 2\pi k\sigma\int_{s=0}^{s=a}\frac{z_p\hat{z}}{\left[s^2+z_p^2\right]^{\frac{3}{2}}}s\,ds = -2\pi k\sigma\left.\frac{z_p}{\sqrt{s^2+z_p^2}}\right|_{s=0}^{s=a}\hat{z} = -2\pi k\sigma z_p\left[\frac{1}{\sqrt{a^2+z_p^2}}-\frac{1}{\sqrt{z_p^2}}\right]\hat{z}$$

Let's expand this for a large:

$$\frac{1}{\sqrt{a^2+z_p^2}} = \frac{1}{a}\frac{1}{\sqrt{1+\left(\frac{z_p}{a}\right)^2}} \approx \frac{1}{a}\left[1-\frac{1}{2}\left(\frac{z_p}{a}\right)^2+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{z_p}{a}\right)^4+\ldots\right]$$

Now let z_p only be in the positive region. We then have:

$$\vec{E} \approx -2\pi k\sigma z_p\left[\frac{1}{a}-\frac{z_p^2}{2a^3}-\frac{1}{z_p}\right]\hat{z} \approx -2\pi k\sigma\left[\frac{z_p}{a}-1\right]\hat{z}$$

Now it is easy to see that if $a \to \infty$ while z_p remains finite that the field becomes:

$$\vec{E} = 2\pi k\sigma\hat{z} = \frac{2\pi\sigma}{4\pi\varepsilon_0}\hat{z} = \frac{\sigma}{2\varepsilon_0}\hat{z}$$

You can compare this result to that obtained from Gauss's law for an infinite plane to see that both predict that the electric field from an infinite plane is uniform.