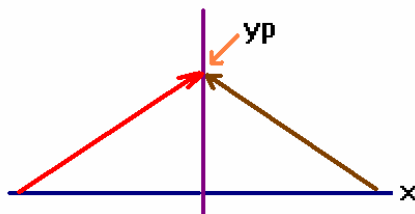


Supplementary Calculus problem for worksheet 2

a line of charge of length $2L$ is aligned symmetrically along the x -axis and has a total charge Q . Find the electric field at points above the wire's central bisector.



I am going to define a charge density λ so that $Q = (2L)\lambda$.

The electric field from a small charge q_i located at x_i along the $+x$ axis is given by:

$$\vec{E}_{p+x} = k \frac{q_i}{x_i^2 + y_p^2} \frac{-x_i \hat{x} + y_p \hat{y}}{\sqrt{x_i^2 + y_p^2}}$$

The electric field from a small charge q_i located at x_i along the $-x$ axis is given by:

$$\vec{E}_{p-x} = k \frac{q_i}{x_i^2 + y_p^2} \frac{x_i \hat{x} + y_p \hat{y}}{\sqrt{x_i^2 + y_p^2}}$$

Let's choose to pick these charges two at a time (one from $+x$ and one from $-x$) and add the electric fields together to get:

$$\vec{E}_p = k \frac{2q_i y_p}{[x_i^2 + y_p^2]^{3/2}} \hat{y}$$

For the non-calculus students, this is as far as can be derived. The calculus students can obtain the final result:

$$\vec{E} = \int_{\text{all charges}} d\vec{E} = 2k \int_{x_i=0}^{x_i=L} \frac{y_p dq}{[x_i^2 + y_p^2]^{3/2}} \hat{y} = 2k\lambda y_p \int_{x_i=0}^{x_i=L} \frac{dx}{[x_i^2 + y_p^2]^{3/2}} \hat{y}$$

This is pretty easy to evaluate if you use the integrator. The required input is:

$$(x^2 + y^2)^{-3/2}$$

And the output is:

$$\frac{x}{y_p^2 \sqrt{x^2 + y_p^2}} \Big|_0^L = \frac{L}{y_p^2 \sqrt{L^2 + y_p^2}}$$

The electric field is then given by:

$$\vec{E} = 2k\lambda y_p \left\{ \frac{L}{y_p^2 \sqrt{L^2 + y_p^2}} \right\} \hat{y}$$

As you get very far from the wire, you would expect this to look like a point charge. Let's confirm that.

$$\vec{E} = 2k\lambda y_p \left\{ \frac{L}{y_p^2 \sqrt{L^2 + y_p^2}} \right\} \hat{y} = \frac{2k\lambda}{|y_p|} \frac{L}{y_p \sqrt{1 + \left(\frac{L}{y_p}\right)^2}} \hat{y} \approx \frac{2k\lambda}{y_p^2} L \hat{y} = \frac{k(2\lambda L)}{y_p^2} \hat{y} = \frac{kQ}{y_p^2} \hat{y}$$

This is exactly the type of behavior that you would expect to see from a point charge.