**Lab 9: Converging and Diverging Lenses**

*revised spring 2007*

In today's lab, you will experiment with the thin lens equation and determine the focal length of converging and diverging lenses.

**Important note: your light source should be located close to zero on the meter stick.**

## Part I: Converging Lenses

I want you to run the OpticsBenchApplet (you'll to use Internet Explorer to do this.).
Set up the following situation:

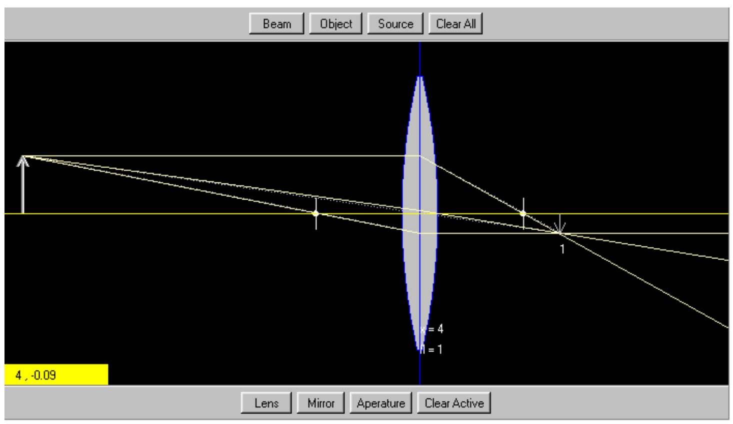
**Lens:**

a converging lens at 2.00
adjust the focal length so one point is at 3 while the other is at 1.

**Object:**

Place a relatively small object at 0.15

Now move the lens to 4.00. Notice where the image is located. You will see that the image is outside the focal length, it is inverted and reduced.




Next I want you to SLOWLY move the lens back towards the object and notice how the motion of the image occurs. Also notice that while the image remains inverted, at one point the motion changes direction ... instead of moving to the right it starts moving to the left. Notice the lens location and the image location where this happens. You can move your mouse over the image to see the result which, if you have done it correctly is about 4.15. This means that the distance from the image to the object is 4 units. The lens is located at about 2.15 units meaning it is about 2 units from the object. Now continue to move the lens towards the object and you should continue to observe the image characteristics. Instead of being reduced, it will be enlarged.

The characterizations of the image are:
(first case): Real: Inverted: Reduced
(second case): Real: Inverted: Enlarged

You will notice that the minimum distance between the image and the object is 4 focal lengths. This would be in general true for the thin lens equation and converging lenses since the units here are rather arbitrary. You are going to observe exactly this behavior in reality in the lab today. Minimize (but don't close) the optics bench applet and start Firefox to access the spreadsheet helper for today's lab. You'll want to open up the spreadsheet helper for today's lab and look at the converging lens data that I took first (then, delete it).

**Warning: The light bulb covers can become hot. Don't touch them!**

You should set up your system in the following way:

Object: at 1.5 cm

Converging lens: close to the object

Screen: at 95 cm

You'll want to cut your lamp on but don't apply more than about 40% of the full scale voltage since this will destroy the lamps.

Now move your converging lens until you obtain an image. Record the location in the spreadsheet under Lens p1. Then move the lens until you obtain the second image. Record your lens location under Lens p2 on the spreadsheet. Both of these locations are recorded beside the "95" screen position. Now continue making measurements for each of the locations that I've indicated on the spreadsheet. You should see the graph of screen location vs. lens location start to fill out (G1). You should particularly notice if any of your measurements have not followed the curve since it is very easy to make an incorrect reading from the meter stick in partial darkness. If you see an incorrect measurement, repeat the measurement rather than discarding it. Do two measurements for each of the screen locations shown on the spreadsheet.

Now I want you to do some analysis on your data before you proceed. You will notice that there are two fit possibilities today for the converging lens. This needs some explanation.

I'll use the following notation: Object position: O: Lens position :L: Screen position S (notice that these are not distances, rather these would correspond to readings directly from the meter stick.)

According to the thin lens equation (with our correct sign convention) we have:

$$\frac{1}{L-O}+\frac{1}{S-L}=\frac{1}{f}$$

This can be written as a quadratic equation as follows:

$$\frac{1}{L-O}+\frac{1}{S-L}=\frac{1}{f}\Rightarrow\frac{S-L+L-O}{(L-O)(S-L)}=\frac{1}{f}\Rightarrow f\left(S-O\right)=LS-OS-LO-L^2$$

Now collect the terms and write it in a standard form:

$$\Rightarrow -L^2+L\left(S-O\right)-f\left(S-O\right)-OS=0\Rightarrow L^2-L\left[S-O\right]+f\left[S-O\right]-OS=0$$

The solutions to the last equation are given by the quadratic formula:

$$L=\frac{[S-O]\pm\sqrt{[S-O]^2+4OS-4f[S-O]}}{2}=\frac{[S-O]\pm\sqrt{[S+O]^2-4f[S-O]}}{2}$$

Now what this says is this: knowing f, S and O, the two lens positions L are solvable.

In your data, you have O, S, and L. You do not have f. This will come from a fit. Fitting the data to multi-valued functions such as this is often prone to failure so I'll show you my approach to the problem: I have rewritten the problem as follows:

$$\left\{2L-\left[S-O\right]\right\}^2=\left[S+O\right]^2-4f\left[S-O\right]$$

I then calculated the right hand side and from all data, fit this function to provide FConvFit1 which is the focal length for the converging lens by fitting the data to the function shown above. You should now do the same by running the solver in excel. You'll need to choose as the target cell H6 and the cell to vary is H4. The focal length then should result. Make sure that you correctly choose the target and change cells.

Now a second method that gave me a somewhat better result was to fit the focal length directly from the thin lens equation. According to the thin lens equation, we have the following:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

Here, we obtain $d_0 = L - O : d_i = S - L$ for each of the lens positions. I've placed all the data into 2 columns on the spreadsheet (look at columns K and L). A plot of $d_i$ as a function of $d_o$ is then shown in graph G2. Next, it is possible to get a theoretical $d_i$ by fitting f a second time to the following function:

$$d_i = \frac{1}{\frac{1}{f}-\frac{1}{d_o}}$$

This is what I did with the second fit. You should do this also. by running the solver with cell H14 as your target cell and the cell to be varied is H12 which is FConvFit2 (Focal Length of the converging lens obtained by the second fit). In my case I obtained a somewhat more pleasing result.

**Part II: Measurement of the focal length of a diverging lens**

Now that you have measured the focal length of the converging lens, I want to show you how to do this with a diverging lens. Open your optics bench.
Move your converging lens until the image appears at 4.5.
**You have two positions that you could choose here.**

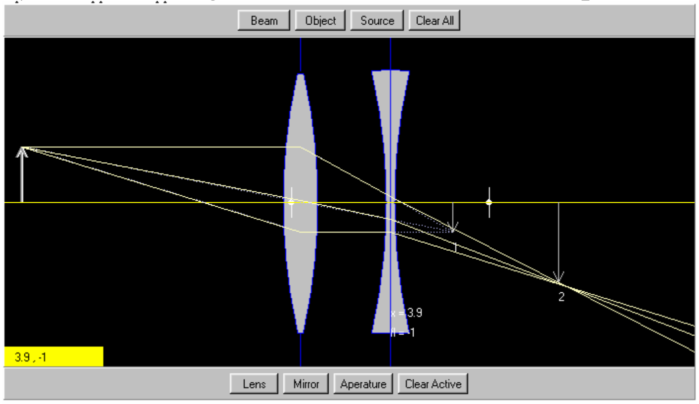
## Choose the converging lens position which is closer to the light source for this portion of the lab.

**If you choose instead the smaller image, you will run into the converging lens and not be able to take all data.**

I'm going to call this image I1. Now you will not move the converging lens again and you know that the image from the converging lens is exactly located at 4.5 units.
Next, insert a diverging lens and adjust the focal length of it until it is -1. You can do this by pulling one of the focal points through the lens. The bottom number should say 1 although seeing the negative sign is not too easy.
Now place the diverging lens between the converging lens and I1 and close to I1. This image will serve as the object for the diverging lens. Since it is behind the lens, it is a virtual object so we can have a real image resulting from this combination. You will observe the image I2 which is inverted from the original object but not inverted from the virtual object. Let me show you how to define the distances:

Diverging Lens position: DL
Virtual object position: 4.5
$d_{o,2}$ =object distance=DL-4.5 (which is negative).

$d_{i,2}$ =second image distance=I2-DL (which is positive).




So long as you do not move the lens so far from the virtual object so that it is outside the focal length of the diverging lens, an image will form. In practice we won’t have a problem with that in lab since our meter stick is only 1 m long.

I would characterize the overall image as {inverted: real: enlarged}.
One further note: although we won’t use this in today’s lab, the final magnification is the image is the product of the individual magnifications:

$$M = M_1 M_2$$

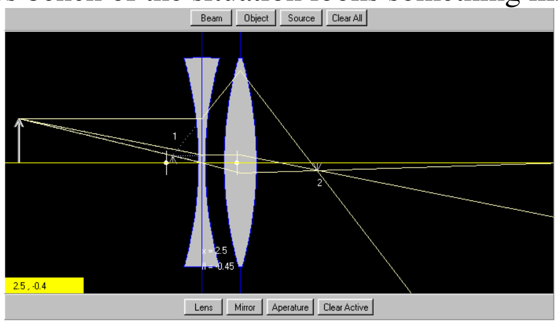
Now I want you to set up a similar situation on the meter stick: place the screen at S=45 cm. Move the convex lens away from the screen towards the light source so that the first (closest) image appears on the screen. The image ought to be reduced, that is how you can know you have the correct image here. You will not adjust the convex lens again in the lab. You must keep the first image exactly at the present location!

Now place the diverging lens between the converging lens and the screen, with it being closer to the screen. Move the screen to S=46. Move the diverging lens until a sharp image forms and record the lens position in the spreadsheet. Repeat these measurements for each of the positions specified in the spreadsheet.  You will want to delete my data before you start entering your own data in the spreadsheet. If you can’t do positions close to the screen, do them as close as you can to start with.

Now I want you to run the solver to fit the thin lens equation for the focal length of the diverging lens.  You do this by choosing H8 as the target cell to minimize while H4 is the cell to be changed. After you run your solver, you should obtain a focal length which is around -10 (the lenses are supposed to have this focal length but, as you can see from my data, I had about 10 % error in this measurement).

And now as a conclusion to this lab, I have a problem for you to work through. Suppose a converging lens makes an image of a distant object at a location 1 cm behind the lens. (a) What is the focal length of the lens? (b) Suppose that you wanted the image to appear actually 1.5 cm behind the first lens and to have the same orientation as the first image. What is the focal length of the lens that you place 1 cm in front of the converging lens to accomplish this?

A sketch from the optics bench of the situation looks something like this:




Here is my solution: The distant object produces an image which is virtual since this is the only type of image a diverging lens can form with a real object. The image location is:

$$\frac{1}{d_o}+\frac{1}{d_i}=-\frac{1}{[f_d]}\Rightarrow d_i=-|f_d|$$

since the object is a distant object.

Now this image will serve as the object for the second (converging) lens. This lens is separated by a distance of 1 cm from the diverging lens and is placed behind the diverging lens. This means that the object distance for the converging lens will be:

$$d_o=1.0+|f_d|$$

(you've got to look at the convention for the proper thing to do here).

Now we know what the focal length of the converging lens is: it is 1 cm and it is positive. We are now ready to set up the thin lens equation that will give us the answer. We want the image from the second lens to appear at 1.5 cm so:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\Rightarrow\frac{1}{1+|f_d|}+\frac{1}{1.5}=\frac{1}{1}\Rightarrow\frac{1}{1+|f_d|}=\frac{1.5-1}{1.5}=\frac{.5}{1.5}=\frac{1}{3}\Rightarrow 1+|f_d|=3\Rightarrow|f_d|=3-1=2$$

So it would seem that a diverging lens with a focal length -2 cm would work here.

Here is my solution .. I need to show it with a beam. See if your answer agrees with mine.