

The charge to mass ratio of the electron

In this experiment, you are going to duplicate a classical experiment of modern physics. Late in the 1800's JJ Thompson was able to provide a measurement of the e/m ratio of the electron by deflecting an electron beam through a magnetic field. Your experiment today is basically the same idea as this first experiment but it has the advantage of 100 years of evolution to become a relatively simple experiment to perform. In the late 1800's however, this experiment was very much at the cutting edge of technology.

Here is how the experiment is supposed to work. A heater voltage is applied to a filament. The filament heats up and thus makes electrons relatively easy to release if a potential difference is applied in this region. The liberated electrons are accelerated through this potential difference to a velocity. We can find this velocity by equating the work to accelerate an electron through a potential difference to the resulting change in kinetic energy:

$$eV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}}$$

Here, e is the charge on the electron, V is the potential difference, m is the mass of the electron and v is the velocity of the electron.

Now, suppose that these electrons are subjected to a magnetic field which is at right angles to the velocity. Then, you know the electron will experience a force given by

$$\vec{F} = e\vec{v} \times \vec{B} \Rightarrow |\vec{F}| = evB$$

Since the force is always at right angles to the velocity, circular motion will result. You may recall from general physics that the acceleration for uniform circular motion is given by:

$$a_c = \frac{v^2}{R}$$

with R being the radius of orbit. Since the Lorentz force produces this acceleration, we can equate the two to find another expression for the velocity:

$$evB = m \frac{v^2}{R} \Rightarrow v = \frac{e}{m} BR$$

But, we know what the velocity of the electron is from the first equation. I will use that to obtain a value for the ratio of the charge to mass for the electron:

$$\sqrt{\frac{2eV}{m}} = \frac{e}{m} BR \Rightarrow \frac{2eV}{m} = \left(\frac{e}{m}\right)^2 B^2 R^2 \Rightarrow \frac{e}{m} = \frac{2V}{B^2 R^2}$$

So, if we can measure V and B and the radius of orbit, we have all the information necessary to measure (e/m)! In the present lab, our magnetic field is produced by Helmholtz coils which are much like a solenoid, only different. An important property of Helmholtz coils, however, is that they produce a uniform magnetic field in their center. It can be shown that the value of this magnetic field is approximately given by

$$B = \frac{(N\mu_0)I}{(5/4)^{3/2}a}$$

where N is the number of turns on a coil (here, N=130), I is the current through the coil, a is the radius of the coils and μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ in SI units).

If we use this magnetic field in our expression for (e/m) we find that the ratio is given by

$$\frac{e}{m} = \frac{2V}{R^2} \frac{(5/4)^3 a^2}{N^2 (\mu_0)^2 I^2}$$

So the procedure is relatively clear now ... measure and record a. Vary (and record) V and I in such a way that the radius of orbit of the electron beam is a nice circle which has the ruler mounted on the back of the apparatus centered across a diameter. (This is why you need to correctly vary both V and I keeping R constant.

Your circle should be centered on about +/- 4 cm. You can determine for yourselves if this should be smaller or larger. The procedure now is to increase your voltage from 0 until you are able to get your circle **and its reflection** to center on the 4 cm marks (or what you have chosen). This last step is very important ... a small amount of parallax will lead to rather large experimental errors here. You will need to vary the current in order to get a perfect fit. Next, increase your voltage from this lowest voltage up to about 350 volts in 20 volt steps. At each 10 volt change, you will change the current in such a way so as to keep the circle centered on the 4 cm marks (or what you have chosen). Record the voltage and current needed at each point. Upon completion, turn your voltage and current down to low levels.

Analysis

If you look back at your expression for the (e/m) ratio, you see that it can be expressed as

$$CI^2 \left(\frac{e}{m} \right) = V$$

where C is a constant given by

$$C = \frac{R^2 N^2 \mu_0^2}{2a^2 (5/4)^3}$$

Here, R is 4 cm (0.04 m or what ever you chose), N is 130, and a is the radius of the Helmholtz coils. Calculate the value of this constant. Now, in your excel spreadsheet, make a column of V and I. Then make a column of I^2/C which is this constant you just calculated. Next, make a plot of $\{I^2/C\}$ vs V. If everything worked perfectly, this would be a straight line with a slope equal to (e/m). Fit a linear fit to your plot and find the slope of this line. The accepted value for the (e/m) ratio is $(e/m)=-1.76 \times 10^{11}$ C/kg. Compare your slope (you can neglect the negative sign in the accepted value if you like) to the accepted value by use of the percentage error. Don't be surprised if you are off somewhat ... after all, the velocity of the electrons must be in error since the only way the gas in the tube can glow is if it is excited by the electrons, and this very action removes kinetic energy from the electrons. You should note that a correction for the Earth's magnetic field is not all that large in this experiment. You can confirm this by looking at the beam deflection when the external magnetic field is completely off.