Equations of motion

Restriction: **Constant** acceleration in the **–y direction only is g**.

In the SI system of units, we use $g = 9.8\frac{m}{s^2}$ and this is positive for the equations below.

General symbols: (x,y)=position (in m)
t=time (in s)
v=speed (in m/s)
g=acceleration due to gravity (=9.8 m/s^2)
θ=angle of velocity above the positive x direction
R=range (in m)

Direction	X	Y
Position; time; (initial parameters)	$x = x_{initial} + v_{initial,x}t$	$y = y_{initial} + v_{initial,y}t - \frac{1}{2}gt^2$
Velocity; time; (initial parameters)		$v_y = v_{initial,y} - gt$
Velocity; Position (initial parameters)		$v_y^2 = v_{initial,y}^2 - 2g(y - y_{initial})$
Average velocity	$v_{average,x} = v_{initial,x}$	$v_{average,y} = \frac{y - y_{initial}}{t} = \frac{1}{2}\left(v_y + v_{initial,y}\right)$
Position; average velocity, time	$x = x_{initial} + v_{average,x}t$	$y = y_{initial} + v_{average,y}t$
Magnitude of initial speed; speed	$v_{initial} = \sqrt{v_{initial,x}^2 + v_{initial,y}^2}; v = \sqrt{v_x^2 + v_y^2}$	
Angle of launch; angle	$\tan\left(\theta_{initial}\right) = \frac{v_{initial,y}}{v_{initial,x}}; \tan\left(\theta\right) = \frac{v_y}{v_x}$	

Range: (R, in m, measured along the x-direction)

(1) Projectile returns to same level from which it was launched

$$R = \frac{v_{initial}^2 \sin(2\theta_{initial})}{g}; \text{maximum values: } R_{max} = \frac{v_{initial}^2}{g} \text{ when } \theta_{initial} = 45^0$$

(2) Projectile launched from a height h above the ground

$$R = \frac{v_{initial}^2 \cos(\theta_{initial})}{g}\left(\sin\left(\theta_{initial}\right) + \sqrt{\sin^2\left(\theta_{initial}\right) + \frac{2gh}{v_{initial}^2}}\right)$$

$$\text{maximum values:} R_{max} = \frac{v_{initial}}{g}\sqrt{v_{initial}^2 + 2gh} \text{ when } \tan\left(\theta_{initial}\right) = \frac{1}{\sqrt{1 + \frac{2gh}{v_{initial}^2}}}$$