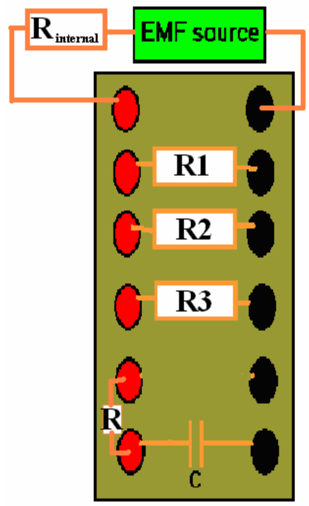
Measurement of the EMF from a source
And the RC time constant of a series RC circuit.

Revised Spring 2007



In this lab, you will explore aspects of the EMF from a battery and you will also investigate the time dependence of the series RC circuit.

Part 1: Measurement of the internal resistance of a battery.

The first part of this lab is intended to help you gain an understanding of the difference between an emf and an potential source. For this experiment, I have provided you with the circuit shown below. The emf source here is a battery which has some internal resistance, which I have added to the normal resistance from a battery. The idea is this: when you place your voltmeter across the open leads of a source of emf, if the voltmeter is ideal, you will not have a current through your circuit. In this configuration, you will measure the emf of your battery, but not the internal resistance that goes along with your battery. Of course, there may be some transient behavior that occurs so the voltmeter reading may slowly build up to a final steady reading for the emf.

You will want to measure and record the values of R1, R2, R3, R and C for future reference at the beginning. You may do this with the voltmeter directly here. (note the placement of R varies slightly from the picture above).

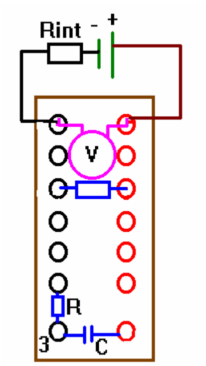


Figure 2

Now, connect your circuit as shown to the left. This configuration allows you to measure the emf (or open-circuit voltage) from your battery. Make this measurement now and record it. It won't be too surprising that the result is around 7 volts (or less, as the batteries age). This is, then, the emf from your source.

All batteries have some internal resistance. In addition, I have placed an external resistance on the battery to add to this internal resistance. Let's see how to measure this and what effect it is going to have on your circuit.

The first connection is shown in figure 2. It consists of the voltmeter measuring directly the emf across the battery. Record this emf below.

EMF from the battery: _____________ volts

While you are at it, you may want to measure the capacitance which is going to be 33μf (you have the option to make this measurement or accept this value).

Capacitance value: ___________ μf

Now we are going to measure the internal resistance. This will be done by measurements of potential drops across the battery when various known resistances are placed in the circuit.

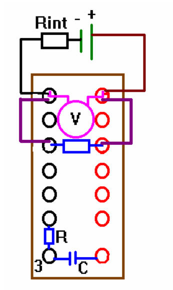


Figure 3

Make the additional connection shown in Figure 3 and observe what happens to the reading on the voltmeter when the final connection is made.

Answer: *the voltage till drop. Why?*

The internal resistance emf and terminal voltage (the terminal voltage is the voltage measured across the terminals of the battery) are related by:

$$V = \mathcal{E} - IR_i$$

When I is zero, you measure directly the emf of the battery. However, you will experience a potential drop due to the internal resistance of the battery once a current is established. Just how this allows a measurement of the internal resistance is worth looking at. You have the emf from your "open circuit" voltage measurement. Now if you make connections that include the ammeter as shown in Figure 4, you will additionally be making a measurement of I. Assuming the emf does not change, you can then solve for the internal resistance:

$$R_{int} = \frac{V - emf}{I}$$

I do advise connecting the voltmeter directly across the resistor as I have shown here.

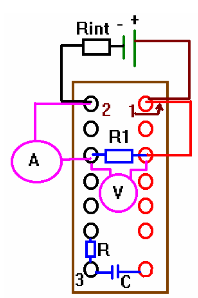


Figure 4

Now that we have two very high impedance instruments we are able to make internal resistance measurements directly. The purpose of R1, R2, R3 and R4 will be simply to vary the current output from the battery. Make your 4 measurements of V and I for each resistor and record the values in the table below. You will want to measure resistance values (particularly for R). This is easily done at the end of your experiment by removing all wire connections, placing the voltmeter across the particular resistor and recording the value. You will need to switch the voltmeter to the "ohms" setting for it to read the resistance.

	Voltage [V]	Current [A]	Resistance [Ohms]
1			
2			
3			
4			

Note: after R2 you should switch the ammeter to the mA scale and leave it there for the rest of the lab today.

You'll want to look at the analysis in the spreadsheet which is included as a spreadsheet. By finding the intercept of your dependence of the internal resistance upon I, you have the internal resistance at I=0 which you can compare to the value of the resistance which is attached to your battery. I am presently not aware of any theory that explains for a given battery the logarithmic dependence upon the current that the internal resistance seems to have: you can regard this as one of those experimental niceties. It might be something that you might want to look more deeply into sometime. It is obtainable in this lab because of the high quality meters that we now have.

To conclude what you have done for the first part of this experiment, you have measured the emf, you measured the terminal voltage and you measured the current for 4 resistors. The internal resistance is related to the terminal voltage by:

$$V = \mathcal{E} - IR_i \Rightarrow R_i = \frac{\mathcal{E} - V}{I}.$$

This permits us to solve for the internal resistance which is done on the spreadsheet.

Part II: The series RC Circuit

Connections for discharge/charge measurements from the RC circuit

(1) You will want to begin by connecting the mx55 which is configured as a voltmeter across the capacitor.

(2) Then connect the mx55 which is configured as an ammeter between the unconnected side of the resistor and the black terminal (or green terminal) connected to the battery.

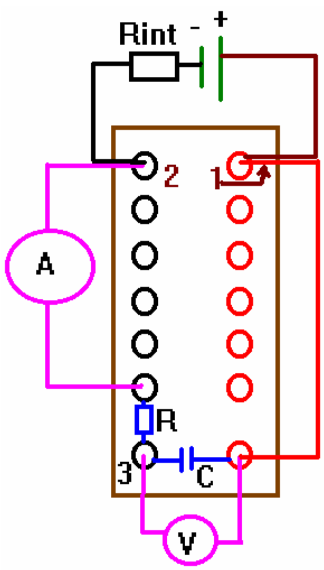


Figure 5

(3) Then connect the red terminal connected to the capacitor to the red terminal across from the black terminal of the battery.
You will need to make sure other connections exist (i.e. from the battery, etc). Never-the-less, when I set up this circuit I usually follow exactly this thought process.

This shows the circuit connection when charging. To make the fully charged circuit discharge, connect the lead marked "1" to the hole marked "2". I have attached an additional black lead to the hole marked "3" so that you are able to fully charge or discharge the capacitor rapidly. You will probably want to try the circuit connections a few times to insure you know how to charge / discharge the circuit.

In making these measurements, make sure your meters are using the 1000 MΩ input impedance, and, of course, the RS232 interface needs to be initialized.

The running of the data acquisition program should be straight forward. You'll name your file, press s to start and t to terminate. The computer will collect data about every 1/3 sec or so. After you press t for terminate, the program will write the data to the data file on the I drive. This may take a few minutes depending upon factors beyond my control. In any event, let the program close itself out ... don't close it out prematurely since you may not get all your data written properly. At the present time, the program is limited to a maximum acquisition of 3500 data tuples.

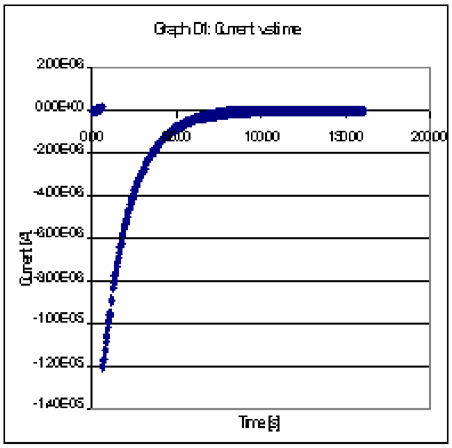
After you know how to charge and discharge your circuit, you should, with a fully charged capacitor acquire data out to about 500 seconds or so. The program will automatically stop after 3500 measurements. You might just want to start it and then (slowly) move away and take a break for about 5 minutes but ... 3500 measurements is a lot of data to work with. You do not want to mess with the circuit while measurements are going on.

Next, you'll acquire data on the charging capacitor (write this to a different data file than the discharging system).

Procedure for analyzing discharge data

Copy your data into the discharge page of the RC spreadsheet.

(1) Determine an acceptable region for your data. You can do this by plotting your current data vs time. Here is my plot.



Locate t0:

t0 is located by looking at the position where the discontinuity in current happened in time.

Scroll down through your data from the beginning to locate this time. Some of my data looks like that represented below. Clearly my t_0 happened between 6.203 s and 6.535 s.

On the spreadsheet I used t_{before} as 6.203 s and t_{after} as 6.535s.

Enter your two times to determine an average value which will be t_0. I have highlighted important times with yellow here. You will notice that I actually did not choose the moments that the current deviated from zero here. This is because It took me a certain amount of time to make the connection. When I moved the cable, the instrument detected the motion of the cables.

Time [S]	current [A]	voltage [V]
0.164	0.00E+00	6.44E+00
...	...	...
5.605	9.00E-08	6.40E+00
5.906	1.00E-07	6.39E+00
6.203	1.00E-07	6.39E+00
6.535	-1.20E-05	6.38E+00
6.844	-1.18E-05	6.24E+00
7.148	-1.17E-05	6.05E+00
7.445	-1.17E-05	5.98E+00
7.715	-1.13E-05	5.98E+00

Determination of an acceptable range of data

Next you are ready to look at a logarithmic plot of your data and determine an acceptable region for fit. The discharge current is negative but it behaves logarithmically. The functional dependence is given by:

$$I(t) = -I_{max} e^{-\frac{t-t_0}{\tau}} : I_{max} = \frac{emf}{R}$$

Now I am going to follow our usual approach with this data. In order to determine the time constant, the form of I must be written slightly differently:

$$\ln[abs(I)] = \ln[abs(I_{max})] - \frac{t-t_0}{\tau}$$

Now we'll define the plotting variables:

$$y \equiv \ln[abs(I)] : b \equiv \ln[abs(I_{max})] : m \equiv \frac{1}{\tau} : x \equiv t - t_0$$

Then the discharge current will have the form:

$$y = mx + b$$

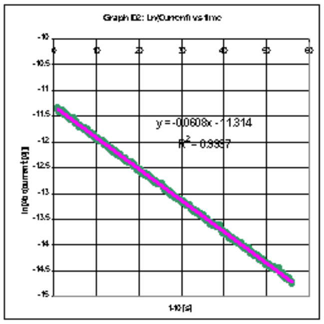
The slope will be negative so the RC time constant will be given by:

$$RC = -m = \frac{1}{\tau}$$

and the maximum current will be given by:

$$I_{max} = e^b$$

When you fit your data, however, you will need to select an acceptable region for the fit. Probably you will need to choose points a few seconds after your t0. You will enter the first and last rows of validity in the spreadsheet to plot the acceptable region of data. You can get an idea of the acceptable range by increasing my provided values a bit. From the current graph, you should obtain a slope and an intercept. Enter these values into the spreadsheet to calculate the RC time constant and Imax a second way. You can compare these values to the values obtained previously by way of % deviation.



Here is a picture of my fit. You may want to set the maximum y value a bit lower so that the graph fills the area a bit more (by default, the max would be zero which I reset to -10).

Note: if your original data contained zeros you will want to delete that. Occasionally transmission errors may occur or the meters may experience a temporary overload. My software does not yet correctly detect and remove such points. You will notice on my original discharge spreadsheet that I deleted one such “bad” data tuple. You will need to delete all 6 columns for things to look nine on the plots.

Now we obtained also data regarding the potential difference across the capacitor. This data is plotted in graph 3. According to the theoretical treatment, the potential across the capacitor varies (when discharging) as:

$$V = V_{max} e^{-\frac{t}{\tau}}$$

Here the maximum voltage would correspond to the emf of your batter which you obtained earlier. If we treat this in the same manner as for the current:

$$\ln(V) = \ln(V_{max}) - \tfrac{t}{\tau} : y \equiv \ln(V) : b \equiv \ln(V_{max}) : m \equiv -\tfrac{1}{\tau} : x \equiv t$$

Then a plot of y vs. x should again give a straight line with intercepts and slopes as shown. From the slope of intercept of graph D3, you should be able to obtain the emf of your battery : this slope is the natural logarithm of the battery’s emf which you measured earlier. You’ll notice that the slope of D3 is pretty close to the slope of D2 if you did your experiment correctly. Enter the intercept of D3 into your spreadsheet to get a % deviation between the two measurements.

What the analysis of I and V indicate is that if a plot were made of I vs. V, we should also have a linear relationship:

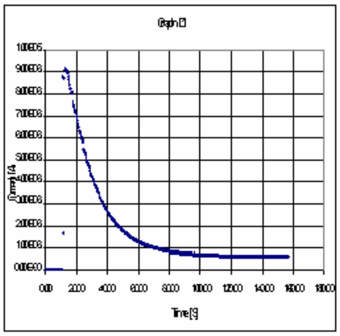
$$\frac{I}{V} = \frac{-I_{max}}{V_{max}} \frac{e^{-\frac{t}{\tau}}}{e^{-\frac{t}{\tau}}} = \frac{-I_{max}}{V_{max}} \Rightarrow V = \left[\frac{V_{max}}{-I_{max}}\right] I = \left[\frac{I_{max} R}{-I_{max}}\right] I = -RI$$

Thus this plot should intersect the origin and the slope of this ought to be the negative of the resistance of the circuit. Place the slope of graph 4 into the appropriate cell to calculate the resistance of your circuit. It should not be too surprising to you that the values are so close since this is essentially the same type of experiment that you did last week with Ohm’s law.

Procedure for analyzing charging data

Analysis of the charging data is not nearly as clean as for discharging data. This is due to several factors including the fact that the terminal voltage from the battery varies throughout the time the experiment is run. For this reason, for the charging data, we will only slightly proceed into the analysis.

We are now going to use a similar procedure to analyze the RC charging data. Following a suggestion from Daniel, I've included the charging data as an additional worksheet.



You will need to copy your RC charging data to the RC-charging page of the spreadsheet as before (in columns A and B). You will obtain a current vs time curve that looks somewhat like what I have shown to the left. You can see that the determination of t0 is not so cleanly defined as previously. Let's ignore the few points leading from zero up to the peak. These are probably attributed to the finite response time of the mx55 ... the measurements can not happen instantly particularly when you are using the 1G input impedance as we are doing in today's lab. But, although you will ignore these points in your analysis, you will still want to use these to determine t0 for discharging. Look through your current data to determine the time at which the current began to change from zero. In my data, at this time I had data that looked like the excerpt shown to the right. Clearly a time before is 10.820 and the time after is 12.113. I suggest actually that the instant right before the connection was made was, in fact 11.453 because of some lag time in my completion of the connection. In any event, these are the times that I'll use as my before and after times in the t0 calculation. Probably when you do your experiment you will want to make these connections quicker than I did. You will want to record these in the discharge spreadsheet which will take the average value to obtain t0. The uncertainty in t0 will increase the error in Imax slightly but not the slope.

Time [S]	current [A]	voltage [V]
0.137	0.00000E+00	0.00000E+00
...	...	...
10.488	0.00000E+00	1.70000E-03
10.820	0.00000E+00	1.70000E-03
11.148	8.77000E-06	7.26000E-02
11.453	1.69000E-06	1.58300E-01
11.785	8.73000E-06	2.33300E-01
12.113	9.09000E-06	3.16000E-01
12.418	9.09000E-06	4.02000E-01
12.750	9.09000E-06	5.02800E-01
13.047	9.16000E-06	5.88400E-01

Theoretically, we expect the following behavior for the current upon charging:

$$I = I_{max} e^{-\frac{t}{\tau}}$$

Now I am going to follow our usual approach with this data. In order to determine the time constant, the form of I must be written slightly differently:

$$\ln\left[(I)\right] = \ln[(I_{max})] - \frac{t - t_0}{\tau}$$

Now we'll define the plotting variables:

$$y \equiv \ln\left[(I)\right] : b \equiv \ln[(I_{max})] : m \equiv \frac{1}{\tau} : x \equiv t - t_0$$

Then the discharge current will have the form:

$$y = mx + b$$

The slope will be negative (as before, the magnitude of the current decreases) so the RC time constant will be given by:

$$RC = m = \frac{1}{\tau}$$

and the maximum current will be given by:

$$I_{max} = e^{b}$$

When you fit your data, however, you will need to select an acceptable region for the fit. Probably you will need to choose points a few seconds after your t0. You should observe that for relatively short times, the behavior of the current is correctly described by the decaying exponential. This is evidenced by the close agreement with a linear fit to the log of current. You can, however, confirm that although the time constant is a bit different, the intercept which represents the log of the maximum current is about the same as for charging. The charging voltage across the capacitor is described by:

$$V = V_{max} - V_{max} e^{-\frac{t}{\tau}}$$

If the emf of the battery replaces the maximum voltage, we then have:

$$emf - V = emf \left[e^{=\frac{t}{\tau}} \right]$$

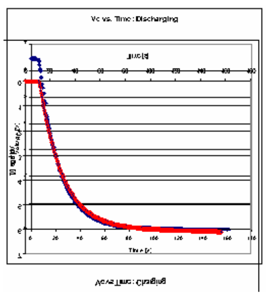
We can transform this by taking the logarithm: $\ln[emf - V] = \ln[emf] - \frac{t}{\tau}$

Now define the following variables:

$$y \equiv \ln[emf - V] : m \equiv -\frac{1}{\tau} : b \equiv \ln[emf] : x \equiv t$$

Then this has the form: $y = mx + b$

Again a linear fit confirms the behavior although the time constant here is quite different from the time constant for the current discharge. This reflects the more complex circuit: the internal effects of the battery are significant in this circuit. You can, however, confirm that the intercept which corresponds to the log of the emf is about the same as we saw with the discharging circuit.



Here, I have plotted on the same scale the discharge voltage (blue) and the charge voltage (red) after flipping it and providing an offset in time. It is clear that the behavior of the two operations is nearly identical although there are differences that show up at short times and also at long times. The actual time-series plots looks like what I am showing below. In Graphs 5 and 6 you will be able to compare your voltage and current data between discharging and charging. It is, however, important to note that these have not been shifted in time to correct for t0 so your plots may be shifted significantly.