

## Image formation in multiple lens systems

The essence of this applications of the thin lens equation to multiple thin lens systems is:

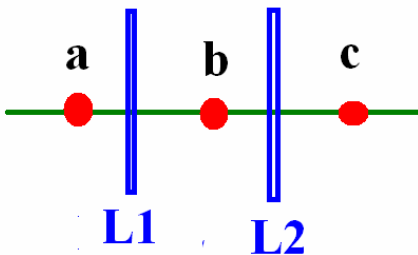
The image from the first lens serves as the object for the second lens.  
You also need to take into account here the possibility that the two lenses are separated.

Here is an example: Consider the following two lenses

Lens1:  $f_1$  : located at  $d_1$

Lens2:  $f_2$  : located at  $d_2$

The separation between the two lenses is quite important here. Let's locate the first lens at 0 and let the second lens be at  $d_2$  behind the first lens.



We have many possibilities that can happen here. The image from lens 1 can be (if it forms) at any of the red points marked above. This image then serves as the object for the second lens. If it occurs at c, then it will become a virtual object for the second lens; otherwise it will be a real object. You also need to worry about the distance that the image from the first lens is from the second lens. We can find out each of these possibilities and it is not all that hard to do. But I really recommend that this is easier to work out on a case by case basis, except is one exceptional situation, that of two thin lenses in direct contact.

### Two thin lenses in direct contact

In this particular case, there is no distance between the two lenses so in fact, we have that the image from the first lens is the object for the second lens and the magnitude of the image distance is the same as the magnitude of the second object distance. This leads to an important result. For any two lenses in direct contact, the effective focal length is:

$$\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

You can prove this by exhausting all the cases for converging and diverging lenses. I'll show you one example: two diverging lenses:

From the first lens, we have:

$$\frac{1}{s_1} + \frac{1}{s_1} = \frac{1}{f_1} \Rightarrow \frac{1}{s_1} = \frac{1}{f_1} - \frac{1}{s_1}$$

For diverging lenses, we know then that the image formed is virtual. This in turn will be a real object for the second lens. We thus find:

$$-\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f_2} \Rightarrow \frac{1}{s_1} + \frac{1}{s_2} - \frac{1}{f_1} = \frac{1}{f_2} \Rightarrow \frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{\text{eff}}}$$

Which says that for 2 diverging lenses in direct contact, the effective focal length is given by  $\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2}$

### An example for two lens not in direct contact

I think that for the more general case of multiple lens systems, it is best to work out the ultimate image on a case by case basis. Let me show you a rather specific example.

Suppose a converging lens (L1) with a focal length of 10 cm is separated by 10 cm from a diverging lens (L2) also with a focal length of 10 cm. Where does the ultimate image form if an object is placed at 15 cm. What is its magnification?

We can find the first image location:  $\frac{1}{s_1} = \frac{1}{+10} - \frac{1}{15} = \frac{3-2}{30} = +\frac{1}{30} \Rightarrow s_1' = +30\text{ cm}$

According to our convention, this image forms behind the lens. The magnification of the resulting image is  $M_1 = -\frac{s_1'}{s_1} = -\frac{+30}{15} = -2$ . This means the image is inverted. If the original object height was 1 cm, the image would have a height of 2 cm.

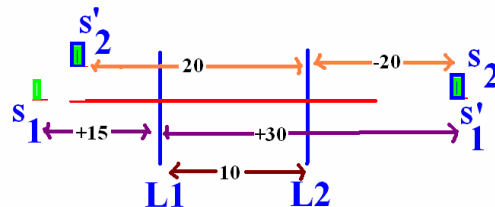
Now the second lens is 5 cm behind the first lens. The object distance is going to be given by  $s_2 = -[30 - 10] = -20\text{ cm}$ . This will be a virtual object for the second lens. We find then that application of the thin lens equation again gives:

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow \frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} = \frac{1}{-10} - \frac{1}{-20} = \frac{-2+1}{20} = -\frac{1}{20} \Rightarrow s_2' = -20\text{ cm}$$

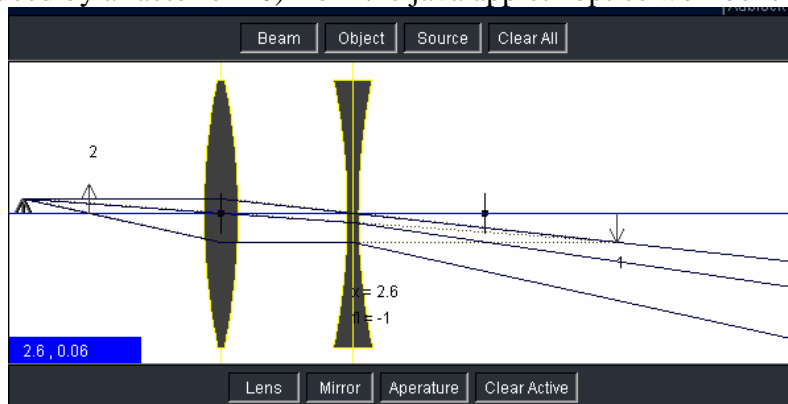
This tells us that the second image is 20 cm in front of the second lens. The magnification of the second image is  $M_2 = -\frac{s_2'}{s_2} = -\frac{-20}{-20} = -1$

The ultimate magnification is given by  $M = M_1 M_2 = +2$

We need to do one final thing: we need to reference the second image to the first image. It's probably best to draw a quick sketch here:



The ultimate image is located 10 cm to the left of lens L1 and thus is a virtual image overall. It is characterized as [Virtual: Upright: Magnified]. Here is a capture of this problem (reduced by a factor of 10) from the java applet "optics workbench":



The ultimate image is labeled "2"