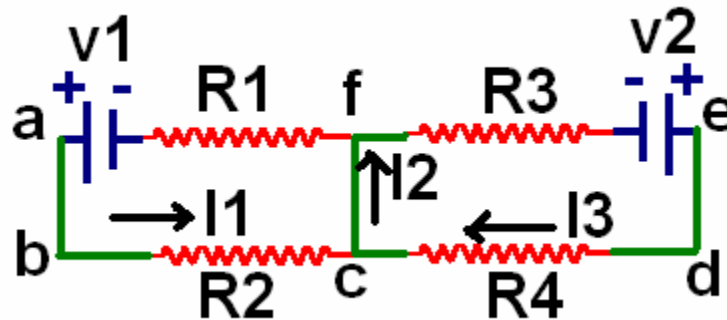


Consider the circuit shown below. Find the currents in each branch.



We have then 2 loops and 1 junction. Kirchoff's laws say:

$$(abcfa) : v_1 - I_1 R_1 - I_1 R_2 = 0$$

$$(edcfe) : v_2 - I_3 R_4 - I_3 R_3 = 0$$

$$(@c) : I_1 + I_3 - I_2 = 0$$

$$v_1 - I_1 R_1 - I_1 R_2 = 0$$

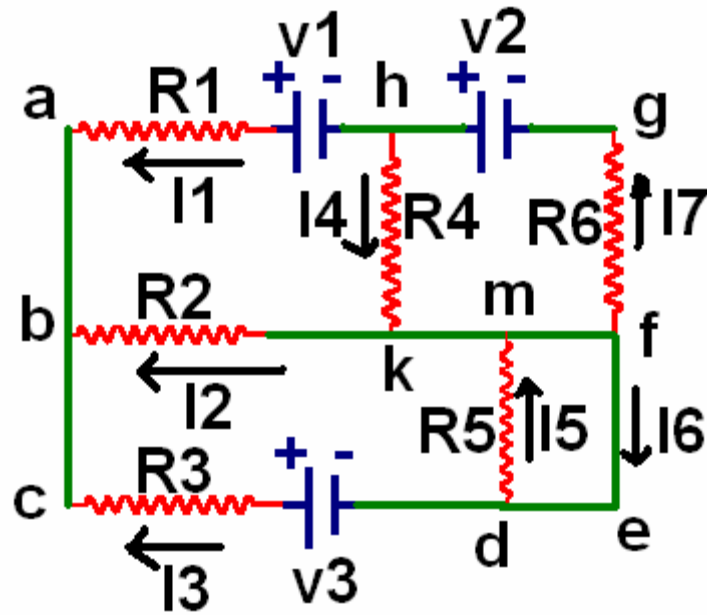
$$v_2 - I_3 R_4 - I_3 R_3 = 0$$

$$I_1 + I_3 - I_2 = 0$$

Solutions for currents:

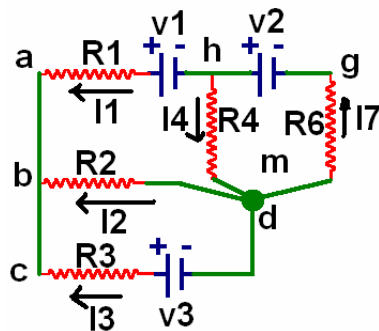
$$I_1 = \frac{v_1}{R_1 + R_2} ; I_2 = \frac{v_1}{R_1 + R_2} + \frac{v_2}{R_3 + R_4} ; I_3 = \frac{v_2}{R_3 + R_4}$$

The answer should not be too much of a surprise.

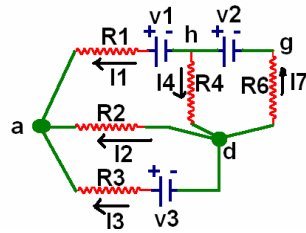


Needed: 4 loops, 3 junctions  
 (abkha):  $I_2 \cdot R_2 + I_4 \cdot R_4 + V_1 - I_1 \cdot R_1 = 0$   
 (bcdmb):  $I_3 \cdot R_3 - V_3 - I_5 \cdot R_5 - I_2 \cdot R_2 = 0$   
 (hkfgh):  $-I_4 \cdot R_4 - I_7 \cdot R_6 + V_2 = 0$   
 (defmd):  $I_5 \cdot R_5 = 0$   
 @h:  $I_7 - I_1 - I_4 = 0$   
 @b:  $I_1 + I_2 + I_3 = 0$   
 @kmf:  $I_5 - I_2 - I_6 + I_4 - I_7 = 0$

You can note that the single solution here for  $I_5$  is zero. This is because the wire (defm) shorts out  $R_5$ . The circuit is therefore can be simplified. In a simpler form, it looks like this:



We can also simplify the section (abc) so that the circuit looks like this:



Here, we need 3 loops and 2 junctions:

$$(ada): +I_3 R_3 - V_3 - I_2 R_2 = 0$$

$$(adha): I_2 R_2 + I_4 R_4 + V_1 - I_1 R_1 = 0$$

$$(hdgh): -I_4 R_4 - I_7 R_6 + V_2 = 0$$

$$@a: I_1 + I_2 + I_3 = 0$$

$$@d: I_4 - I_2 - I_7 - I_3 = 0$$

If you solve these, the results are fairly nasty.

What if, however, you now asked a question such as: supposing all resistors are the same, what value of  $v_3$  would cause  $I_1$  to become zero?

I'll assign a value of  $R_4$  to each of these resistors. The result is this:

$$I_1 = \frac{2V_1 + V_2 - V_3}{4R_4}$$

$$I_7 = \frac{2V_1 + 5V_2 - V_3}{8R_4}$$

$$I_3 = \frac{-2V_1 - V_2 + 5V_3}{8R_4}$$

$$I_4 = \frac{-2V_1 + 3V_2 + V_3}{8R_4}$$

$$I_2 = \frac{-2V_1 - V_2 - 3V_3}{8R_4}$$

Looking at the solution for  $I_1$  you would then require:

$$2v_1 + v_2 - v_3 = 0 \Rightarrow v_1 = \frac{v_3 - v_2}{2}$$

Checking this result with the solver site, we have:

$$I_1 = 0$$

$$I_7 = \frac{v_2}{2R_4}$$

$$I_3 = \frac{v_3}{2R_4}$$

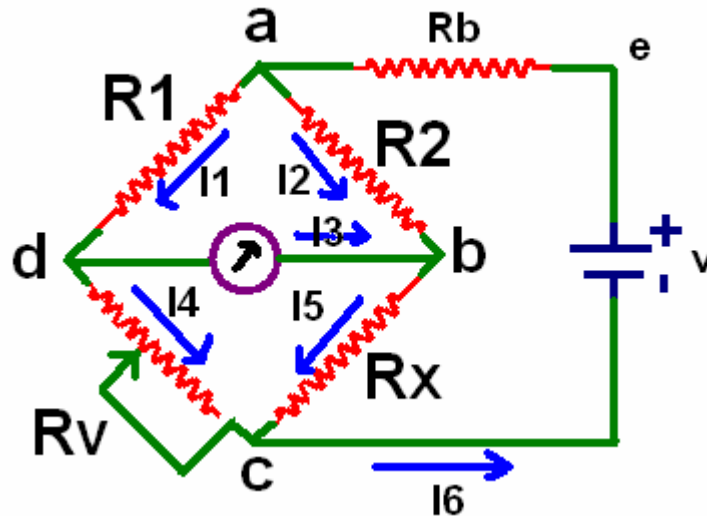
$$I_4 = \frac{v_2}{2R_4}$$

$$I_2 = -\frac{v_3}{2R_4}$$

Notice for this particular case, we would have  $I_3 = -I_2$  as is required. Also note that  $I_4$  and  $I_7$  are also the same. Essentially we have broken the circuit into two branches.

You can now find the power dissipated in each element of the circuit and the total power dissipated.

**The Wheatstone Bridge Circuit:**  
Measurement of the unknown resistance  $R_x$ .



When the bridge is balanced,  $I_3$  is zero which means there is no potential difference between d and b.

$$(adcea): -I_1 R_1 - I_4 R_v + V - I_6 R_b = 0$$

$$(abcea): -I_2 R_2 - I_5 R_x + V - I_6 R_b = 0$$

$$(adba): -I_1 R_1 - I_3 R + I_2 R_2 = 0$$

$$@a: I_6 - I_1 - I_2 = 0$$

$$@c: I_4 + I_5 - I_6 = 0$$

When entering these into the solver, use  $x$  for  $R_x$ ,  $s$  for  $R_v$  and  $b$  for  $R_b$ .

If the bridge is now balanced, then  $I_1 = I_4$  and  $I_2 = I_5$ .

Add these additional two conditions to the solver.

$$I_3 = - \frac{R_1 V x - R_2 s V}{R (b R_1 + R_2 R_1 + x R_1 + b R_2 + b s + R_2 s + b x + s x)}$$

This gives condition for  $I_3$  to be zero:

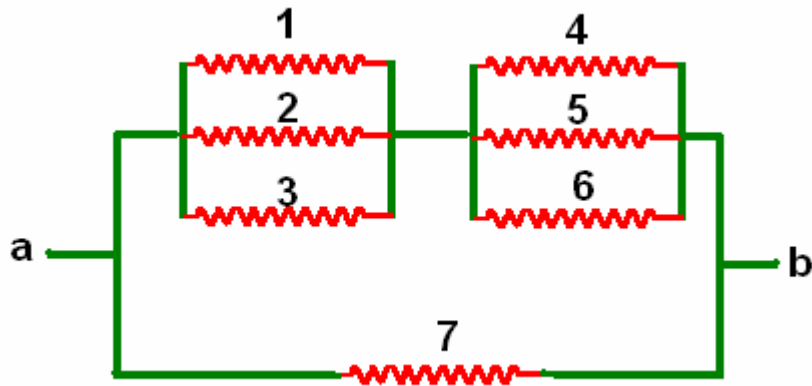
$$V(R_1 R_x - R_2 R_s) = 0$$

From here, it is easy to see that the value of the unknown resistance is given by:

$$R_x = \frac{R_2}{R_1} R_s$$

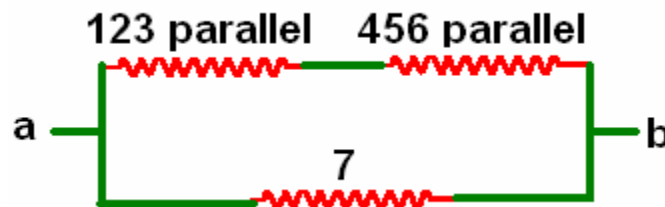
## Reduction of circuits

It is possible with many simple circuits to reduce the circuits without Kirchoff's laws.  
Let me show you some examples of how to do this:

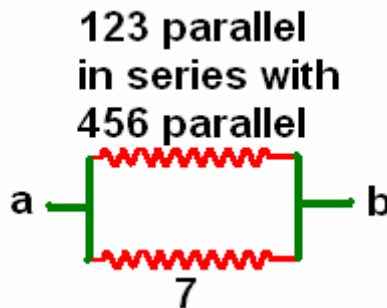


Reduce this circuit in a step-wise method recognizing series and parallel combinations of resistors:

Step 1:



Step 2:



The last step in finding the equivalent resistance is then clear.

If you would like to see a numerical example of this, let the resistors be the value of the number that I've assigned to them. Then

$$123 \text{ parallel} = \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3}} = \frac{1}{\frac{6}{6} + \frac{3}{6} + \frac{2}{6}} = \frac{1}{\frac{11}{6}} = \frac{6}{11}$$

$$456 \text{ parallel} = \frac{1}{\frac{1}{4} + \frac{1}{5} + \frac{1}{6}} = \frac{1}{\frac{30}{120} + \frac{24}{120} + \frac{20}{120}} = \frac{1}{\frac{74}{120}} = \frac{120}{74}$$

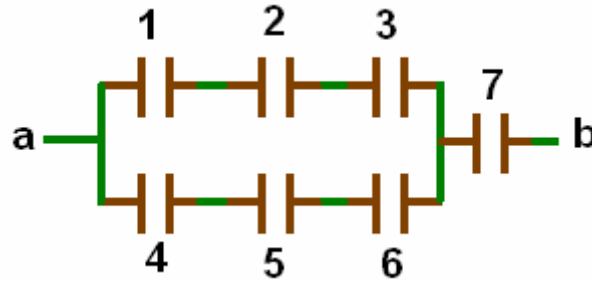
$$(123 \text{ parallel}) \text{ series } (456 \text{ parallel}) = \frac{6}{11} + \frac{120}{74} = \frac{444 + 1320}{814} = \frac{1764}{814}$$

The final step is to calculate the equivalent resistance of the last 2:

$$R_{eq} = \frac{1}{\frac{1764}{814} + \frac{1}{7}} = \frac{1}{\frac{12348 + 814}{5698}} = \frac{5698}{13162} = 0.43\Omega$$

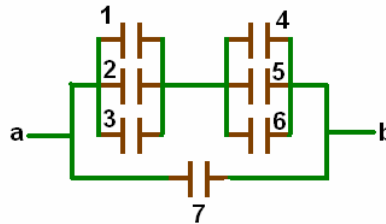
You can work backwards from here to find the current through each resistor. It is somewhat important to be able to recognize these series and parallel combinations of resistors.

The same technique also works for capacitors.  
 in fact, it the same only different .... what is the equivalent capacitance of this circuit?



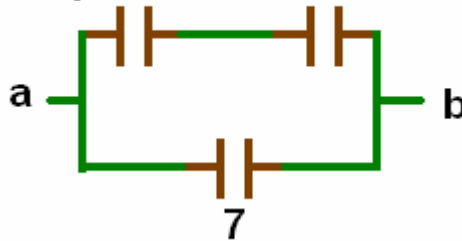
where each of the capacitances is given in  $\mu\text{f}$ ? (without doing further calculations)  
 The answer after you think about it for a while is clearly  $0.43 \mu\text{f}$ .

Let's do the same thing for a similar arrangement of capacitances:



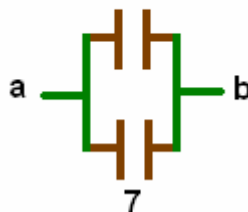
The first reduction is:

**123 parallel 456 parallel**



Then

**123 parallel  
 series  
 456 parallel**



You can calculate this equivalent capacitance:

$$(123 \text{ parallel}) = 1 + 2 + 3 = 6\mu\text{f}$$

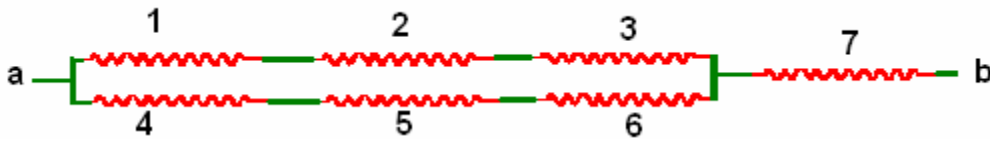
$$(456 \text{ parallel}) = 4 + 5 + 6 = 15\mu\text{f}$$

$$(123 \text{ parallel}) \text{ series } (456 \text{ parallel}) = \frac{1}{\frac{1}{6} + \frac{1}{15}} = \frac{1}{\frac{15+6}{90}} = \frac{90}{21}$$

The equivalent capacitance is then given by:

$$\frac{90}{21} + 7 = 11.29\mu\text{f}$$

Ok, what about this circuit?



Without calculation, if the numbers represent Ohms, then the equivalent resistance is  
 $11.29\Omega$

Unfortunately, these types of identifications (i.e. the connection between capacitive and resistive circuits in the analysis) is of limited utility which is why they're probably not often shown.