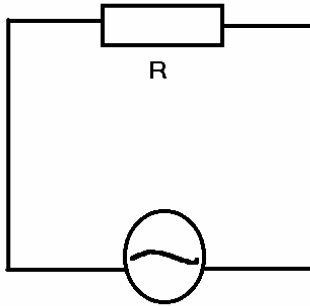


**A note about notation: lowercase  $v$  and  $i$  are instantaneous values.**

$\overleftarrow{v_r}$



$$v = V_m \sin(\omega t)$$

**This analysis is valid if the circuit contains only a resistor and the applied voltage must vary as:**

$$v = V_m \sin(\omega t)$$

The voltage drop across the resistor,  $v_r$  is equal to the instantaneous applied voltage,  $v$ . Thus  $v_r = V_m \sin(\omega t)$ .

The instantaneous current is equal to

$$i = \frac{v_r}{R} = \frac{V_m}{R} \sin(\omega t)$$

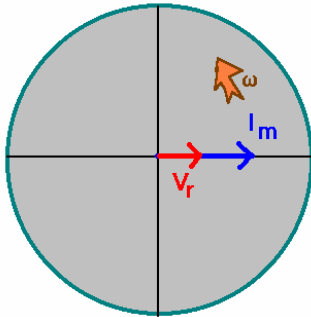
The **peak current** is given by

$$I_m = \frac{V_m}{R}$$

The instantaneous voltage drop across the resistor is

$$v_R = I_m R \sin(\omega t)$$

**and the current is said to be in phase with the voltage.**



We need to calculate the time-average power radiated by the resistor. The instantaneous power radiated by the resistor is given by

$$p = i^2 R$$

where  $i$  is the instantaneous current. We can calculate the time average power by using a slick trick that you have seen me use before. ...

$$\langle P \rangle = \langle i^2 R \rangle = I_m^2 R \langle \sin^2(\omega t) \rangle = \frac{V_m^2}{R} \langle \sin^2(\omega t) \rangle$$

To evaluate the time average of  $\langle \sin^2(\omega t) \rangle$ , consider the trigonometric relation:  $\sin^2\theta + \cos^2\theta = 1$ . The time average of 1 is 1:  $\langle 1 \rangle = 1$ , and the  $\sin^2\theta$  behaves exactly like the  $\cos^2\theta$  and so by symmetry, both of these must contribute equally to the time average. Thus we reach the conclusion that  $\langle \sin^2(\omega t) \rangle = \frac{1}{2}$ . Thus,

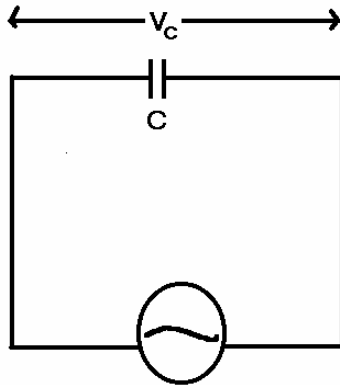
$$\langle P \rangle = \frac{V_m^2}{2R} = \frac{I_m^2 R}{2}$$

We know also that the instantaneous power is also given by  $p = iv$ . For AC circuits satisfying these conditions, we redefine  $I$  to be  $I_{\text{rms}}$  and  $V$  to be  $V_{\text{rms}}$  by

$$I_{\text{rms}} \equiv \frac{I_m}{\sqrt{2}} \quad \text{and} \quad V_{\text{rms}} \equiv \frac{V_m}{\sqrt{2}} \quad \text{so we can write} \quad \langle P \rangle = I_{\text{RMS}} V_{\text{RMS}}$$

**Note this is true only for the initial assumptions in this problem!!!!**

**In particular if we have capacitance and inductance in the circuit, this must be modified by a factor known as the "power factor" which comes later.**



*This analysis is valid if the circuit contains only a capacitor and the applied voltage must vary as:*

$$v = v_m \sin(\omega t)$$

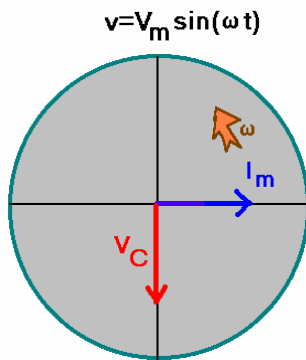
The instantaneous voltage drop across the capacitor is  $v_c = V_m \sin(\omega t)$  The instantaneous current is then

$$i_c = \omega C V_m \cos(\omega t)$$

Notice that the current is **not in phase** with the applied voltage. We can show this most clearly by rewriting the current as

$$i_c = \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The current reaches its peak value 1/4 of a cycle sooner than the voltage reaches its peak value. For this reason, in this type of circuit obeying the initial assumptions it is said that the **current always leads the voltage across a capacitor by 90°**.



The peak current in the circuit is given by  $I_m = \omega C V_m$ .

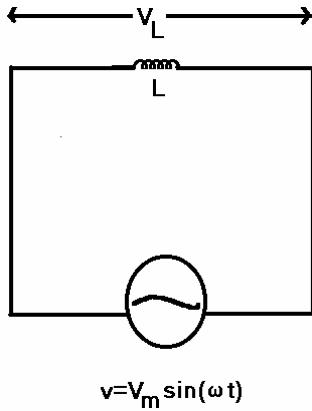
We want to define a new quantity called the **capacitive reactance**:

$$X_c \equiv \frac{1}{\omega C}$$

so that the instantaneous voltage drop across the capacitor is given by

$$v_c = I_m X_c \sin(\omega t)$$

Note that the units of capacitive reactance are those of Ohms! You can show this since  $c=Q/V$  and  $\omega$  has units of "rad"/s so  $[X_c]=[V/I]=\text{Ohms}$ .



*This analysis is valid if the circuit contains only an inductor and the applied voltage must vary as:*

$$v = v_m \sin(\omega t)$$

The instantaneous voltage drop across the inductor is

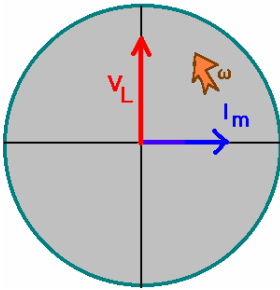
$$v - L \frac{di}{dt} = 0$$

The instantaneous current is then given by

$$i_L = -\frac{V_m}{\omega L} \cos(\omega t)$$

We can rewrite this in terms of the sin as

$$i_L = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$



The current reaches its peak value 1/4 of a cycle later than the voltage reaches its peak value. For this reason, in this type of circuit obeying the initial assumptions it is said that the *current always lags the voltage across an inductor by 90°*.

The peak current through the inductor is given by

$$I_m = \frac{V_m}{\omega L}$$

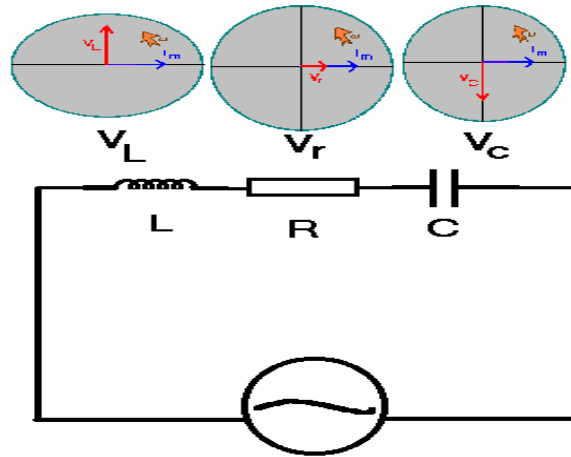
We want to define a new quantity called the *inductive reactance*:

$$X_L \equiv \omega L$$

so that the instantaneous voltage drop across the inductor is given by

$$v_L = I_m X_L \sin(\omega t)$$

Note that the units of inductive reactance are those of Ohms! You can show this since  $[L] = [Vt/A]$  and  $\omega$  has units of "rad"/s so  $[X_L] = [V/I] = \text{Ohms}$ .



$$v = V_m \sin(\omega t)$$

*This analysis is strictly valid ONLY for the series RLC circuit. For other configurations, you must consider other methods for combining the reactances.*

Reactance:

$$X_C \equiv \frac{1}{\omega C} \quad X_L \equiv \omega L$$

Treat the various voltages across the various elements as vectors and find the magnitude:

$$|\vec{V}| = |\vec{V}_R + \vec{V}_C + \vec{V}_L| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

In terms of the reactance, then

$$V_R = I_m R \quad V_L = I_m X_L \quad V_C = I_m X_C$$

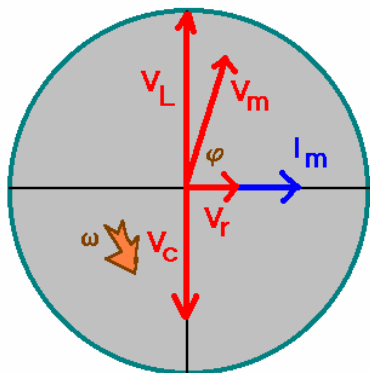
So the magnitude of  $V_m$  is given by

$$|\vec{V}| = V_m = I_m \sqrt{R^2 + (X_L - X_C)^2} = I_m Z$$

*where Z is called the impedance of the circuit*

and the angle that  $V_m$  makes with respect to the current is given by

$$\tan(\phi) = \frac{X_L - X_C}{R}$$



Now you can easily see that the time average power radiated by the RLC circuit is given by

$$\langle P \rangle = I_{RMS} V_{RMS} \cos(\phi)$$

where the last term is the "power factor". You will also notice that resonance occurs when  $X_C = X_L$ . This then gives the resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$

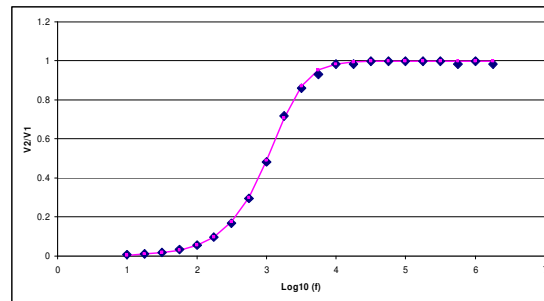
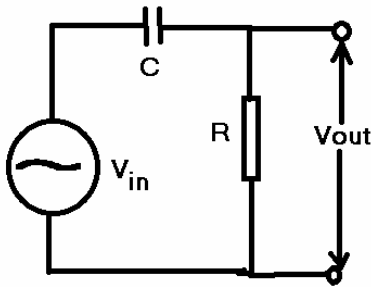
**2 filter circuits: The high-pass filter and the low pass filter.**

The easiest way to remember how to treat RC filter circuits is to look at the capacitive reactance,  $X_c$ . For low frequencies, capacitor acts like an open circuit. For high frequencies, the capacitor acts like a short circuit. We'll get the intermediate response here also.

1. The high-pass filter. We want to look for the ratio of  $V_{out}$  to  $V_{in}$ . This is given by the high-pass *response function*:

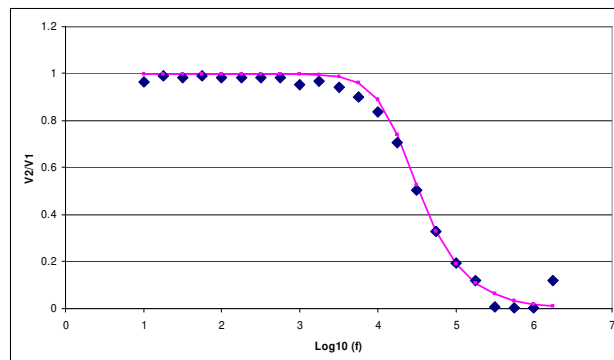
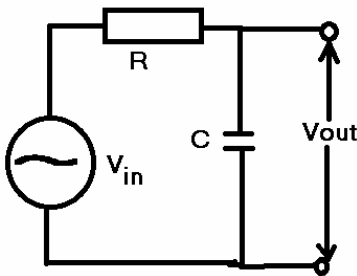
$$\frac{V_{out}}{V_{in}} = \frac{I_m R}{I_m Z} = \frac{R}{\sqrt{R^2 + X_c^2}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}$$

Notice that the HF response is that which would happen for the capacitor replaced by a piece of wire.



2. The low-pass filter. We want to look for the ratio of  $V_{out}$  to  $V_{in}$ . This is given by the low-pass *response function*:

$$\frac{V_{out}}{V_{in}} = \frac{I_m X_c}{I_m Z} = \frac{\left(\frac{1}{\omega C}\right)}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$



Note that at high frequencies, this acts much like a short across the capacitor.

Now a question for you: suppose you replace the capacitor in these two circuits with an inductor. What are the response functions in those cases?