Electrostatic field due to charged thick disk

Charged disk: (height h, radius a, charge density ρ) along symmetry axis.

Charge density: $dq_i = \rho s ds d\varphi dz_i$

$$\vec{r}_i = s\cos\varphi\hat{x} + s\sin\varphi\hat{y} + z_i\hat{z} : \vec{r}_p = z_p\hat{z} : \vec{r}_{ip} = -s\cos\varphi\hat{x} - s\sin\varphi\hat{y} + \left(z_p - z_i\right)\hat{z}$$

$$\vec{E} = \int_{s=0}^{s=a}\int_{\varphi=0}^{\varphi=2\pi}\int_{z_i=0}^{z_i=h} k\rho\frac{-s\cos\varphi\hat{x} - s\sin\varphi\hat{y} + \left(z_p - z_i\right)\hat{z}}{\left[s^2 + \left(z_p - z_i\right)^2\right]^{\frac{3}{2}}} s ds d\varphi dz_i = 2\pi k\rho\int_{s=0}^{s=a}\int_{z_i=0}^{z_i=h}\frac{\left(z_p - z_i\right)\hat{z}}{\left[s^2 + \left(z_p - z_i\right)^2\right]^{\frac{3}{2}}} s ds dz_i$$

$$\vec{E} = 2\pi k\rho\left[\int_{z_i=0}^{z_i=h}\frac{\left(z_p - z_i\right)\hat{z}}{\sqrt{s^2 + \left(z_p - z_i\right)^2}}dz\right]_{s=0}^{s=a} = 2\pi k\rho\left[\int_{z_i=0}^{z_i=h}\frac{\left(z_p - z_i\right)\hat{z}}{\sqrt{a^2 + \left(z_p - z_i\right)^2}}dz - \int_{z_i=0}^{z_i=h}\frac{\left(z_p - z_i\right)\hat{z}}{\sqrt{\left(z_p - z_i\right)^2}}dz\right]$$

$$\vec{E} = 2\pi k\rho\hat{z}\left[-\sqrt{a^2 + \left(z_p - z_i\right)^2}\Big|_{z_i=0}^{z_i=h} - \left[-\sqrt{\left(z_p - z_i\right)^2}\right]_{z_i=0}^{z_i=h}\right]$$

$$\vec{E} = 2\pi k\rho\hat{z}\left[-\sqrt{a^2 + \left(z_p - h\right)^2} + \sqrt{a^2 + z_p^2} - \left[-\sqrt{\left(z_p - h\right)^2} + \sqrt{z_p^2}\right]\right]$$

$$\vec{E} = 2\pi k\rho\hat{z}\left[-\sqrt{a^2 + \left(z_p - h\right)^2} + \sqrt{a^2 + z_p^2} + \sqrt{\left(z_p - h\right)^2} - \sqrt{z_p^2}\right]$$

This can have many signs and values depending upon the relative values of z and h and also upon the absolute value of z.