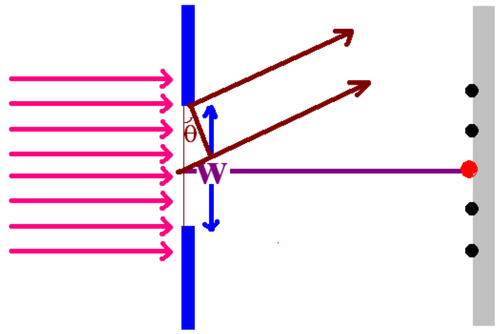
Single Slit diffraction pattern



Our system consists of one single opening which is several wavelengths of light in width. Divide the slit into 2 parts. Then, the path difference is given by:

$$\delta = \pm \frac{W}{2}\sin(\theta)$$

The first bright spot will occur for an angle of 0 degrees. Between this angle and the next bright spot, a dark spot will occur. The angle corresponding to the first dark spot above the central maximum is given by requiring the path difference to be ½ of a wavelength:

$$\delta = \frac{\lambda}{2} \Rightarrow W\sin(\theta) = \pm\lambda$$

We get higher order dark spots in a similar manner: imagine dividing the screen into 4 parts. Then the path difference is given by:

$$\delta = \pm \frac{W}{4}\sin(\theta)$$

The second dark spot will occur when this path difference is ½ of a wavelength:

$$\delta = \frac{\lambda}{2} = \frac{W}{4}\sin(\theta) \Rightarrow W\sin(\theta) = \pm 2\lambda$$

Carrying on with the subdivision, we find that the general condition for the angular locations of dark spots will be given by:

$$W\sin(\theta_m) = \pm m\lambda$$

Now we can use this to find the width of the bright central maximum:
The angular width of the upper portion of the fringe is:

$$\sin(\theta) \approx \theta = \frac{\lambda}{W}$$

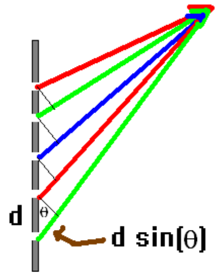
So the total angular width of the bright central maximum is twice this angle or:

$$\Delta\Theta \approx 2\frac{\lambda}{W}$$

If the aperture is circular rather than rectangular, then the width of this central maximum would be:

$$\Delta\Theta \approx 1.22\frac{\lambda}{W}$$

We will use this expression to obtain the resolving power of a lens.



Diffraction grating

Now consider a similar problem but now let the slit be replaced by a grating which a separation between adjacent slits of d. This time, we have many more slits than just one and the slit openings approach the wavelength of light.

The path difference between adjacent slits is given by:

$$d\sin(\theta) = \delta$$

For constructive interference to occur, we need this path difference to be equal to an integer number of wave lengths. Thus:

$$\text{Bright} \Rightarrow m\lambda = d\sin(\theta); m = 0, \pm 1, \pm 2, \ldots$$

You can then see the angular distribution of bright spots:

$$\sin(\theta_m) = \frac{m\lambda}{d}$$

You also notice that there is a maximum order possible. The maximum order is given by:

$$m' = \frac{d}{\lambda}$$

Where m' is the integer smaller than this maximum order. This serves to limit the total number of maxima observed from a diffraction grating.

A very subtle point is now important to realize: the condition for destructive interference for a single slit pattern is the condition for constructive interference for a diffraction grating (but, of course d and W have different meanings.

You can use this to obtain the wavelength of light in a very easy way. Suppose a screen is placed a distance L behind the grating. Then the distance between any two adjacent dots is given by:

$$y_{m+1} - y_m = L\left[\tan(\theta_{m+1}) - \tan(\theta_m)\right]$$

Here, the particular angles are given by:

$$\sin(\theta_{m+1}) = \frac{(m+1)\lambda}{d} \text{ and } \sin(\theta_m) = \frac{(m)\lambda}{d}$$

If the angles are not too big, it is also appropriate to make small angle approximations. In such a situation we can say:

$$\Delta y = L\frac{\lambda}{d} \Rightarrow \lambda = \frac{d\Delta y}{L}$$

But strictly said, this will only work for relatively small angles.

Resolving power of a lens

The Rayleigh Criteria for the minimum resolving power of a lens is what is considered to be the theoretical limit to the resolvability of a lens. It comes by looking at the single slit interference pattern for a circular slit. The result was that the central broad maximum has an angular diameter of where w is the diameter of the lens.

$$\Delta\Theta \approx 1.22\frac{\lambda}{w}$$

Here is the interpretation of this in terms of resolvability: two distant point sources of light are just resolvable when the center of the maximum from the first source lies at the minimum of the second source.

As an example, suppose two red lights (700 nm) are 1 m apart. How far can the two lights be and still be resolved through a lens which is 0.5 m in diameter? What about a lens which is 0.01 m in diameter?

The angular separation of the two lights is given by:

$$\Delta\Theta = \frac{1\text{m}}{r} = 1.22\frac{\left(700\text{x}10^{-9}\,\text{m}\right)}{0.5\text{m}} \Rightarrow r = \frac{0.5}{854\text{x}10^{-9}} = 5.85\text{x}10^{5}\,\text{m} = 585\,\text{km}$$

For the smaller lens, we have 50 times less resolving ability. Thus the smaller lens would only be able to resolve this at a distance of about 11 km.

One additional note on this: the wavelength used here refers to the wavelength inside the lens. The human eye has an index of refraction of about 1.3 which would modify this to read:

$$\Delta\Theta \approx 1.22\frac{\left(\frac{\lambda}{n}\right)}{w}$$

The human eye has a diameter of about 0.05 m. Thus we could resolve this situation at:

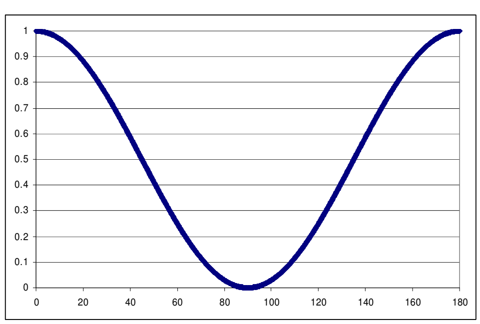
$$\Delta\Theta = \frac{1\text{m}}{r} = 1.22\frac{\left(\frac{700\text{x}10^{-9}\,\text{m}}{1.3}\right)}{0.05\text{m}} \Rightarrow r = 76\ \text{km}$$

You'll notice that this decreases considerably for different wavelengths of light and, of course, atmospheric conditions will ultimately make this limit something that is not actually realized.

Polarizers and the law of Malus

The law of Malus is this: for two crossed polarizers which have an angle θ between their angles of polarization, the transmitted intensity varies as:

$$I = I_0 \cos^2\left(\theta\right)$$



There are many very important properties associated with polarization. One of these which you have already experimented with in lab is Brewster's angle.