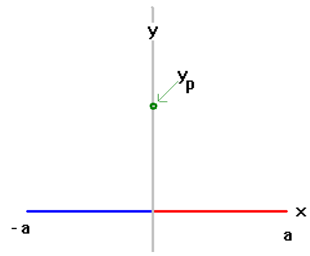
A dipole wire has a total length of 2a and extends from x=-a to x=+a. At x=0, the charge linear distribution changes from -λ to +λ. Find the electric field at points along the symmetric y axis.



$$\vec{E}_p = \int_q \frac{k\,dq}{|\vec{r}_{ip}|^2}\hat{r}_{ip}$$

$$\vec{r}_i = x\hat{x} + 0\hat{y} : \vec{r}_p = 0\hat{x} + y_p\hat{y} : \vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x\hat{x} + y_p\hat{y} : \hat{r}_{ip} = \frac{-x\hat{x}+y_p\hat{y}}{\sqrt{x^2+y_p^2}}$$

$$dq = -\lambda dx\,[-a < x < 0] : dq = \lambda dx[0 < x < a]$$

$$\vec{E}_p = \int_{x=-a}^{x=0} -\frac{k\lambda(-x\hat{x}+y_p\hat{y})}{\left[x^2+y_p^2\right]^{\frac{3}{2}}}dx + \int_{x=0}^{x=a} \frac{k\lambda(-x\hat{x}+y_p\hat{y})}{\left[x^2+y_p^2\right]^{\frac{3}{2}}}dx$$

By symmetry (you can also show by doing the integral) the y component must vanish. Thus:

$$\vec{E}_p = k\lambda\left[\int_{x=-a}^{x=0} \frac{x\hat{x}}{\left[x^2+y_p^2\right]^{\frac{3}{2}}}dx - \int_{x=0}^{x=a} \frac{x\hat{x}}{\left[x^2+y_p^2\right]^{\frac{3}{2}}}dx\right]\hat{x}$$

$$= k\lambda\left[-\frac{1}{\sqrt{x^2+y_p^2}}\Bigg|_{-a}^{0} + \frac{1}{\sqrt{x^2+y_p^2}}\Bigg|_{0}^{a}\right]\hat{x}$$

$$= k\lambda\left[-\frac{1}{|y_p|} - -\frac{1}{\sqrt{a^2+y_p^2}} + \frac{1}{\sqrt{a^2+y_p^2}} - \frac{1}{|y_p|}\right]\hat{x}$$

$$= k\lambda\left[-\frac{1}{|y_p|} + \frac{1}{\sqrt{a^2+y_p^2}} + \frac{1}{\sqrt{a^2+y_p^2}} - \frac{1}{|y_p|}\right]\hat{x} = 2k\lambda\left[\frac{1}{\sqrt{a^2+y_p^2}} - \frac{1}{|y_p|}\right]\hat{x}$$

$$\int \frac{x}{(x^2+y^2)^{3/2}}\,dx =$$

$$-\frac{1}{\sqrt{x^2+y^2}}$$