	Physics 250: Worksheet 3A	Name: ________________

Supplementary problems for electrostatics

1. Three identical charges each with q=1μC are arranged at the vertices of an equilateral triangle with a side of length 1m. Find the magnitude and direction of the vector electric field at the center (where the bisectors intersect). Arrange your triangle so that two vertices are along the x-axis and the third vertex is along the +y axis.

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

4. A sphere of radius a has a charge density which varies as $\rho = Q\frac{r}{a}$ inside and zero outside the sphere. Find the electric field inside and outside the sphere.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

	Physics 250: Worksheet 3A	Name: ________________

1. Three identical charges each with q=1μC are arranged at the vertices of an equilateral triangle with a side of length 1m. Find the magnitude and direction of the vector electric field at the center (where the bisectors intersect). Arrange your triangle so that two vertices are along the x-axis and the third vertex is along the +y axis.

Solution: we first need to find out the coordinates of the center of the triangle. By symmetry the intersection would have the coordinates of (0,.5). We thus have the specifications:
(p:0,.5),(q1:-.5,0),(q2:+.5,0),(q3:0,+1)
Form each of the vectors:

$$\vec{r}_p = 0\hat{x} + .5\hat{y} : \vec{r}_1 = -.5\hat{x} + 0\hat{y} : \vec{r}_2 = .5\hat{x} + 0\hat{y} : \vec{r}_3 = 0\hat{x} + 1\hat{y}$$

Form the differences:

$$\vec{r}_{1p} = \vec{r}_p - \vec{r}_1 = (0 + .5)\,\hat{x} + (.5 - 0)\,\hat{y} = .5\hat{x} + .5\hat{y}$$

$$\vec{r}_{2p} = \vec{r}_p - \vec{r}_2 = (0 - .5)\,\hat{x} + (.5 - 0)\,\hat{y} = -.5\hat{x} + .5\hat{y}$$

$$\vec{r}_{3p} = \vec{r}_p - \vec{r}_3 = (0 - 0)\,\hat{x} + (.5 - 1)\,\hat{y} = 0\hat{x} - .5\hat{y}$$

Form the magnitudes:

$$\left|\vec{r}_{1p}\right| = \sqrt{.5^2 + .5^2} = .5\sqrt{2} : \left|\vec{r}_{2p}\right| = \left|\vec{r}_{1p}\right| : \left|\vec{r}_{3p}\right| = .5$$

Form the unit vectors:

$$\hat{r}_{1p} = \frac{\vec{r}_{1p}}{\left|\vec{r}_{1p}\right|} = \frac{.5\hat{x} + .5\hat{y}}{.5\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y} : \hat{r}_{2p} = -\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y} : \hat{r}_{3p} = 0\hat{x} - 1\hat{y}$$

Write down the electric field at p:

$$\vec{E}_p = kq\left\{\frac{\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}}{.5} + \frac{-\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}}{.5} + \frac{0\hat{x} - 1\hat{y}}{.5^2}\right\} = kq\{2.828 - 4\}\,\hat{y}$$

$$= kq\{0\hat{x} - 0.586\hat{y}\} = (0\hat{x} - 10532\hat{y})\,\tfrac{N}{C}$$

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.
Form each of the vectors:

$$\vec{r}_{1p} = \hat{x} + \hat{y} : \vec{r}_{2p} = -\hat{x} + \hat{y} : \vec{r}_{3p} = -\hat{x} - \hat{y} : \vec{r}_{4p} = \hat{x} - \hat{y}$$

Form the magnitudes:

$$\left|\vec{r}_{1p}\right| = \left|\vec{r}_{2p}\right| = \left|\vec{r}_{3p}\right| = \left|\vec{r}_{4p}\right| = \sqrt{2}$$

Form the unit vectors:

$$\hat{r}_{1p} = \frac{\hat{x} + \hat{y}}{\sqrt{2}} : \hat{r}_{2p} = \frac{-\hat{x} + \hat{y}}{\sqrt{2}} : \hat{r}_{3p} = \frac{-\hat{x} - \hat{y}}{\sqrt{2}} : \hat{r}_{4p} = \frac{\hat{x} - \hat{y}}{\sqrt{2}}$$

Form E:

$$\vec{E}_p = \frac{k\mu}{2\sqrt{2}}\left\{(1 - 2 - 3 + 4)\,\hat{x} + (1 + 2 - 3 - 4)\,\hat{y}\right\} = \frac{k\mu}{2\sqrt{2}}\{0\hat{x} - 4\hat{y}\} = 12713\,\tfrac{N}{C}$$

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.
In this case, only charges 1 and 2 contribute to fields along the x direction and only charge 3 contributes to the field along the y direction. The electric field vectors are easily written:

$$\vec{r}_{1p} = \hat{x} : \vec{r}_{2p} = -\hat{x} : \vec{r}_{3p} = -\vec{y} : \left|\vec{r}_{1p}\right| = \left|\vec{r}_{2p}\right| = \left|\vec{r}_{3p}\right| = 1 : \hat{r}_{1p} = \hat{x} : \hat{r}_{2p} = \hat{y} : \hat{r}_{3p} = \hat{y}$$

Then the electric field is given by:

$$\vec{E}_p = k\mu\{-\hat{x} - \hat{x} - \hat{y}\} = k\mu[-2\hat{x} - \hat{y}] = \{-71980\hat{x} - 8990\hat{y}\}\tfrac{N}{C}$$

4. A sphere of radius a has a charge density which varies as $\rho = Q\frac{r}{a}$ inside and zero outside the sphere. Find the electric field inside and outside the sphere.

$$Q_{enc} = \iiint \rho dV = \tfrac{4\pi Q_0}{a}\int_{r=0}^{r=a} r \bullet r^2 dr = \tfrac{\pi Q_0}{a} a^4 = \pi a^3 Q_0$$

The electric flux is given by: $\Phi = \oint \vec{E} \bullet d\vec{A} = E\oint d\vec{A} = E4\pi r^2$

By Gauss's law, the electric field is then given by:

$$\vec{E} = \tfrac{\pi a^3 Q_0}{4\pi\varepsilon_0 r^2}\hat{r} = \tfrac{Q_0 a^3}{4\varepsilon_0 r^2}\hat{r}$$

Inside the sphere, the charge enclosed is given by:

$$Q_{enc} = \iiint \rho dV = \tfrac{4\pi Q_0}{a}\int_{r=0}^{r=r} r \bullet r^2 dr = \tfrac{\pi Q_0}{a} r^4$$

So the electric field inside the sphere is given by:

$$\vec{E} = \frac{\frac{\pi Q_0 r^4}{a}}{4\pi\varepsilon_0 r^2}\hat{r} = \tfrac{Q_0}{4\varepsilon_0}\tfrac{r^2}{a}\hat{r}$$

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

The surface charge density is given by $\sigma = \frac{Q}{A} = 1\frac{\mu C}{m^2}$

Choosing a cylindrical Gaussian surface of area A' we have that between the plates, the electric flux through the ends of the cylinder is given by:

$$2EA' = \tfrac{\sigma A'}{\varepsilon_0} \Rightarrow E = \tfrac{\sigma}{2\varepsilon_0} = \tfrac{1x10^{-6}}{2(8.85x10^{-12})} = 5.65x10^4\,\tfrac{N}{C}$$